

Chapter

# 9

## Shear Force and Moment Diagrams

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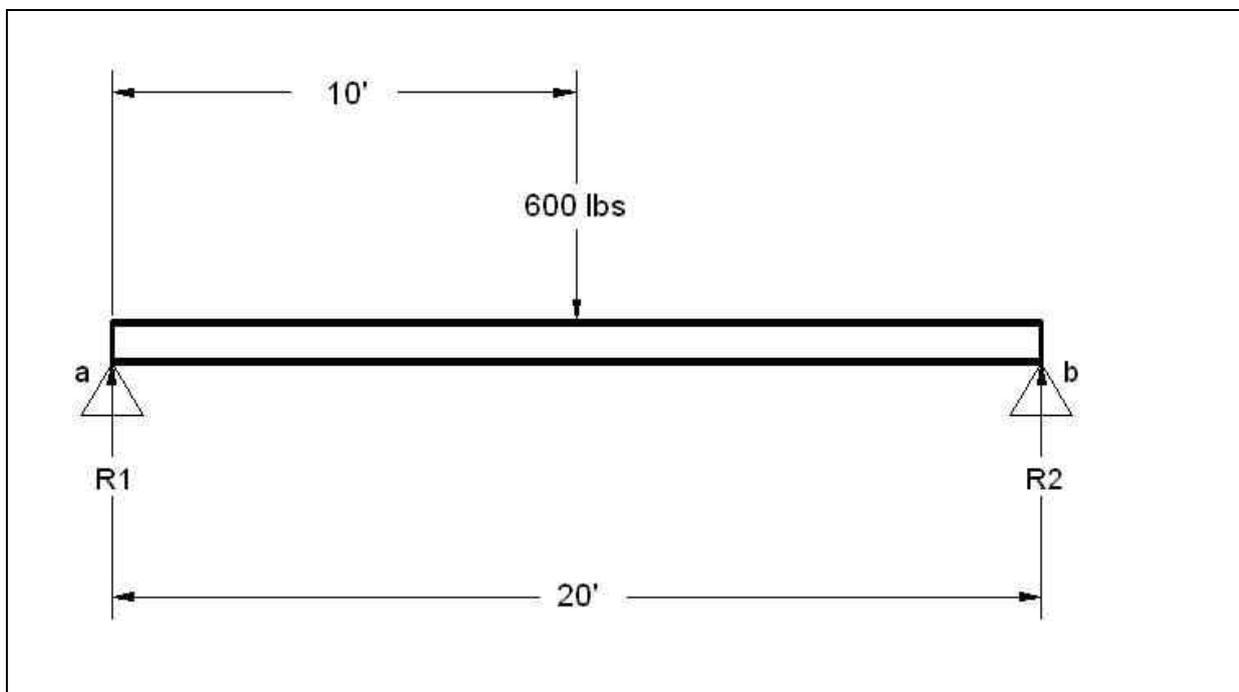
**In this chapter, you will learn the following to World Class standards:**

- **Making a Shear Force Diagram**
- **Simple Shear Force Diagram Practice Problems**
- **More Complex Shear Force Diagrams with Multiple Loads**
- **Complex Shear Force Diagram Practice Problems**
- **Drawing a Shear Force Diagram for a Simple Cantilever Truss**
- **Drawing a Bending Moment Diagram for a Simple Truss**
- **Draw Free Body, Shear Force and Bending Moment Diagrams**

## Making a Shear Force Diagram

To determine the point where the supported truss is most prone to breakage, we use a shear force diagram to analyze the beam. The diagram starts at the zero point on both the left and right ends of the graph and when we see the data line cross the zero point somewhere in between the shear diagram, we see the point where the beam is weak. Designers and engineers need to examine the cross sectional area and the material at that point of the beam to determine if the truss is strong enough to bear the assessed load. For CAD specialist, the Shear Force diagram is another type of drawing that can help the engineer analyze an assembly.

Before making the shear force or the moment diagram



**Figure 9.1 – Simple Truss**

When we solve a simple supported truss, we will examine the problem in equilibrium. In the problem in Figure 9.1, we sum the forces in the Y – direction, with the weight of 600 pounds acting in a positive direction and the two reactionary forces R1 and R2 repulsing the weight with in a negative path. With just this equation, we are unable to solve for the unknowns. We need to write a second formula, the sum of the moments where we scrutinize the forces acting around support A. Both equations are shown below:

$$\Sigma F_y = 600 \text{ lbs} - R1 - R2 = 0$$

$$\Sigma M_a = (600 \text{ lbs} \times 10 \text{ ft}) - (R2 \times 20 \text{ ft}) = 0$$

The sum of the moment around support A is the 600 lbs times 10 feet or 6000 ft – lbs. Divide the 6000 ft – lbs by 20 feet and we see the answer for R2 as 300 pounds.

$$R2 \times 20 \text{ ft} = 6000 \text{ ft} - \text{lbs}$$

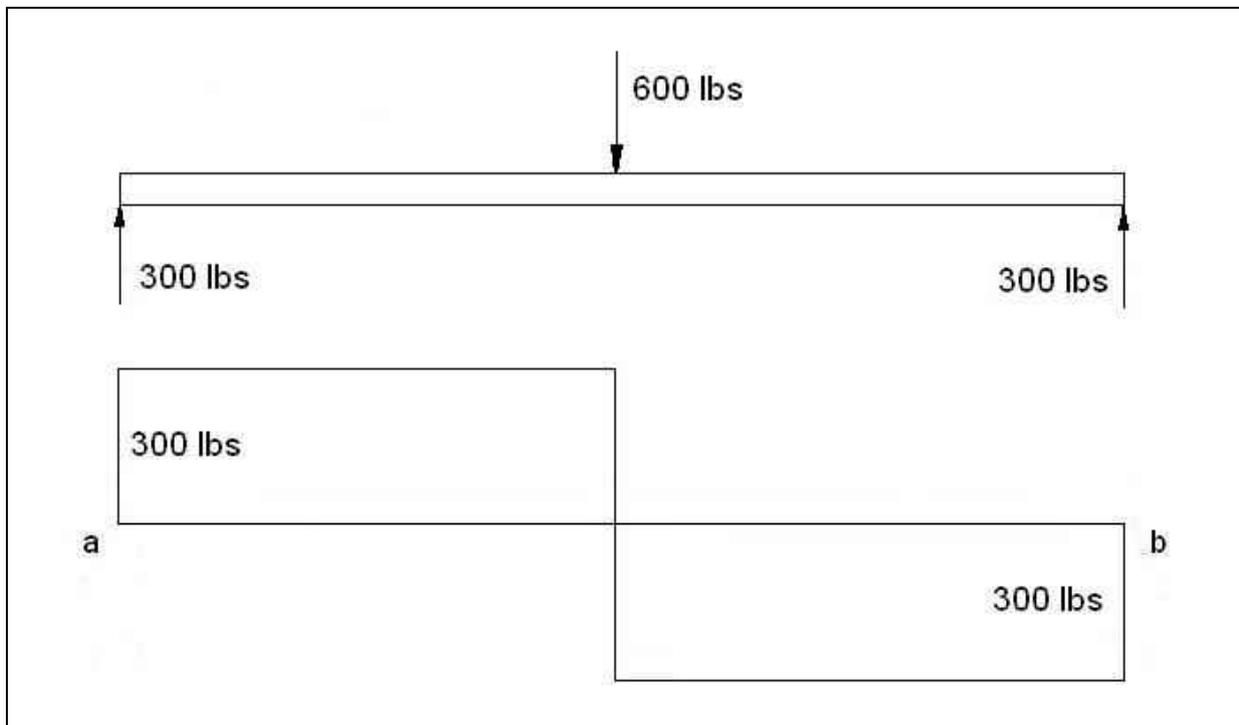
$$R2 = 6000 \text{ ft} - \text{lbs} \div 20 \text{ ft} = 300 \text{ lbs}$$

We use the equation for the sum of forces in the Y – direction to find R1, by plugging in the value of R2 into the formula. As shown below, we see the amount for R1 is 300 lbs.

$$R1 = 600 \text{ lbs} - R2$$

$$R1 = 600 \text{ lbs} - 300 \text{ lbs} = 300 \text{ lbs}$$

Below the beam in the CAD file, draw a 20-foot long line. Draw a line on the left end of the 20-foot long line that is 30 units high for the 300 pound force at R1. We are making each unit represent 10 lbs of force. Draw another line to the right 10 feet (120 inches). Draw the next line down 60 units for the 600 lb force. Draw the next line to the right 10 feet (120 inches). The last line goes up 30 units high for the 300 pound force at R2. We now observe our first shear force diagram.



**Figure 9.2 – Simple Shear Force Diagram**

Whenever we make a truss diagram and solve for the unknown units, we now have the capability to draw a Shear Force Diagram directly below the first layout. By lining up the two diagrams, we can easily apply the changes in the Shear Force diagram in the correct locations. Many designers find the task easier, if they can use projection lines from the free body diagram on the top to draw the vertical changes on the Shear Force diagram. Always properly label the drawing and save the file in the project folder for future reference.

Next, we have a series of practice problems to reinforce our methods.

## Simple Shear Force Diagram Practice Problems

In our CAD program, draw the truss to scale, placing the weight in the correct position. Solve for the unknown reactive forces. Draw a Shear Force diagram below each truss layout.

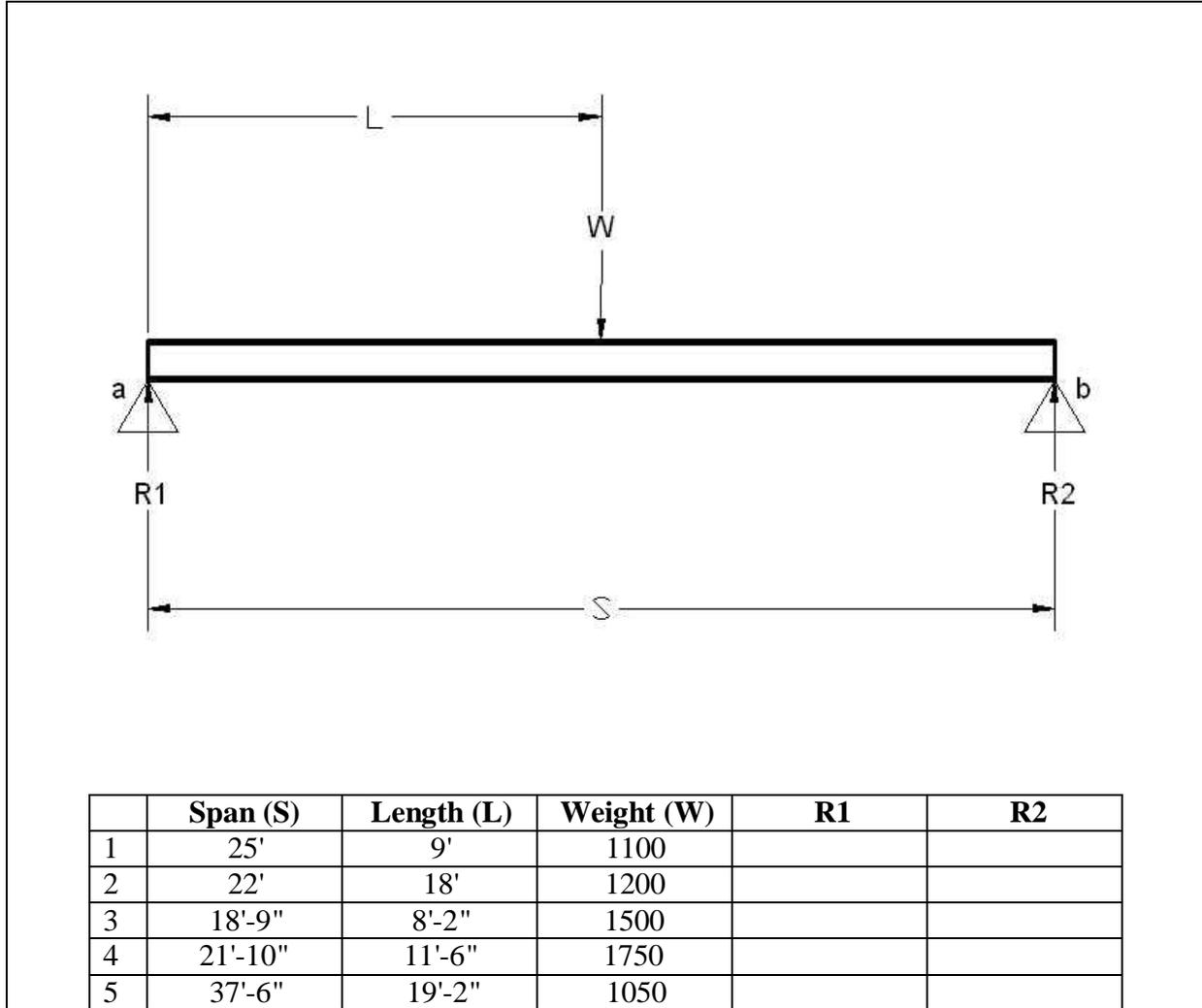
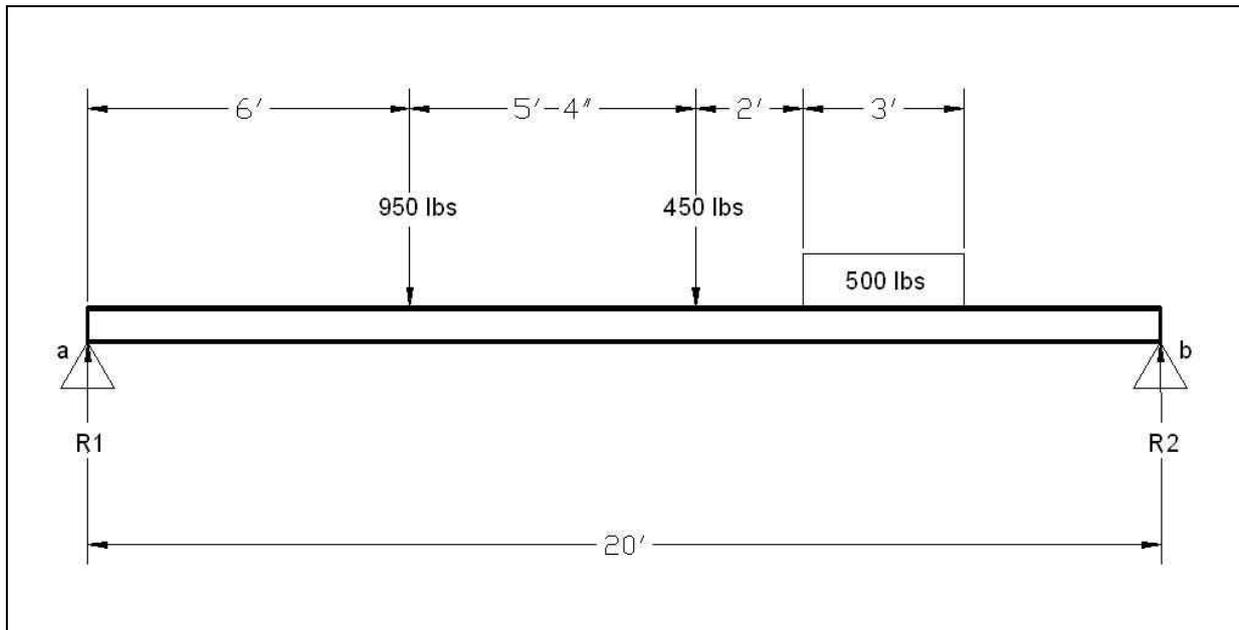


Figure 9.3 – Simple Truss Practice Problems

## More Complex Shear Force Diagrams with Multiple Loads

As in the previous chapter, trusses and beams can have multiple loads acting upon the structure. When there is more than one load, the Shear Force diagram can become more interesting to draw and interpret. In figure 9.3, we have four loads on the beam, where one of the load is not a point load, but is spread over the length of the beam that supports the weight.



**Figure 9.3 – Simple Supported Truss with More Than One Load**

In the first formula, we add the three loads which are 950 pounds, 450 pounds and 500 pounds and subtract reactive forces at the supports R1 and R2. In the second equation, we have clockwise moments of 950 lbs × 6 ft, 450 lbs × 11.333 ft, and 500 lbs × 14.833 feet. We measured to the center of gravity of the 500 pound weight to get 14.833 feet. The last moment is in a counterclockwise direction and is R2 × 20 ft.

$$\Sigma F_y = 950 \text{ lbs} + 450 \text{ lbs} + 500 \text{ lbs} - R1 - R2 = 0$$

$$\Sigma M_a = (950 \text{ lbs} \times 6 \text{ ft}) + (450 \text{ lbs} \times 11.333 \text{ ft}) + (500 \text{ lbs} \times 14.833 \text{ ft}) - (R2 \times 20 \text{ ft}) = 0$$

First, we solve the sum of the moment equation, by finding the foot – pounds quantity for each load and then divide by 20 feet to force R2. The value for R2 is 880.81 lbs as shown below.

$$\Sigma M_a = (950 \text{ lbs} \times 6 \text{ ft}) + (450 \text{ lbs} \times 11.333 \text{ ft}) + (500 \text{ lbs} \times 14.833 \text{ ft}) - (R2 \times 20 \text{ ft}) = 0$$

$$R2 = 5700 \text{ ft.} - \text{lbs} + 5099.85 \text{ ft.} - \text{lbs} + 7416.5 \text{ ft.} - \text{lbs} \div 20 \text{ ft.}$$

$$R2 = 17616.2 \text{ ft.} - \text{lbs} \div 20 \text{ ft.} = 880.81 \text{ lbs.}$$

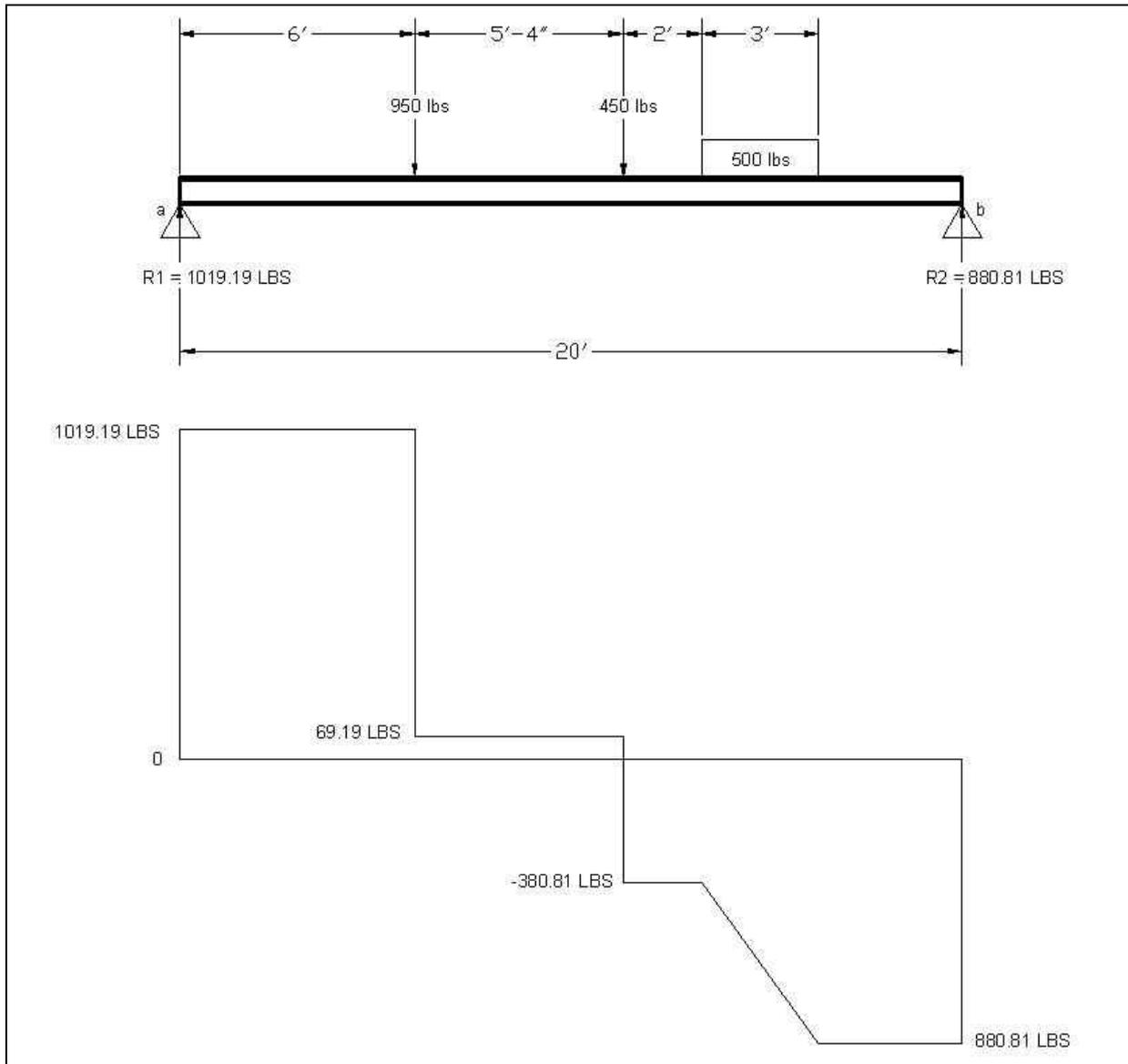
After solving for force R2, we utilize the sum of forces in the Y – direction to find R1, by plugging in the value of R2 into the formula. As shown below, we see the amount for R1 is 1019.19 lbs.

$$\Sigma F_y = 950 \text{ lbs} + 450 \text{ lbs} + 500 \text{ lbs} - R1 - R2 = 0$$

$$\Sigma F_y = 950 \text{ lbs} + 450 \text{ lbs} + 500 \text{ lbs} - R1 - 880.81 = 0$$

$$R1 = 1900 \text{ lbs} - 880.81 \text{ lbs} = 1019.19 \text{ lbs}$$

As you can see in figure 9.4, we draw both the free body diagram and the Shear Force diagram to line up with each other. Whenever there is a change in force on the top layout, we can observe the alteration in the graph below. Just up to the six-foot mark on the left side of the Shear Force graph, the reading is 1019.19 lbs, but at the 6-foot mark, the graph drops 950 lbs to 69.19 lbs with the addition of the first load on the beam. A Shear Force diagram changes abruptly with point loads, however when loads are uniform as the 500 lbs on the right side of the truss, the slope of the graph is at an angle, changing incrementally along the area of contact.



**Figure 9.4 – Shear Force Diagram for Truss with More Than One Load**

Always take the time to label the Shear Force diagram, because we might decide to make a CAD drawing unit equal 10 lbs or even 100 lbs. By indicating the points of change, the designer or engineer who reads your drawing file later will be able to interpret your data correctly.

## Complex Shear Force Diagrams Practice Problems

Figure 9.5 has five problems to compute truss and Shear Force diagrams when each case has three loads. Solve for the unknown reactive forces. Draw a Shear Force diagram below each truss layout.

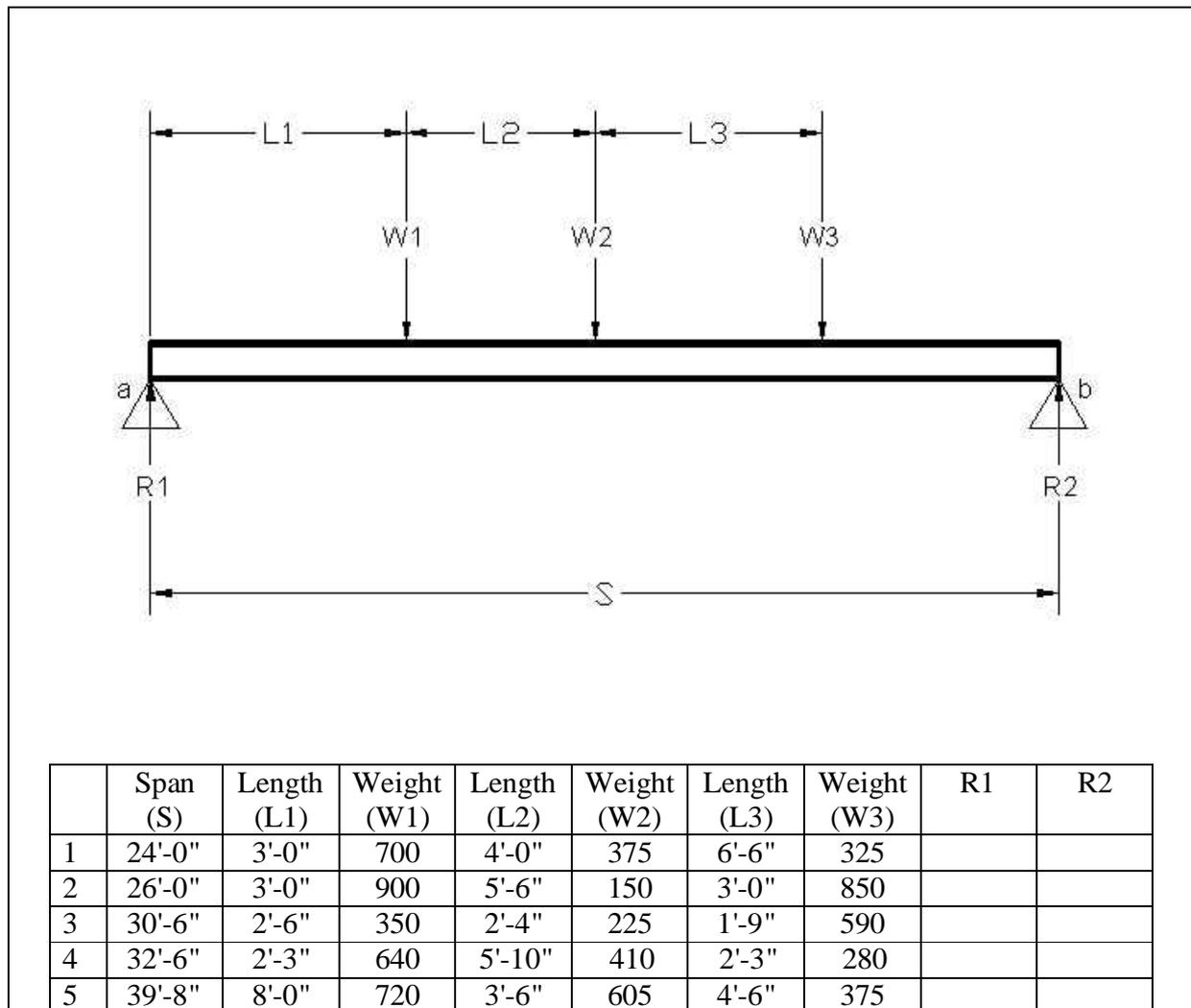
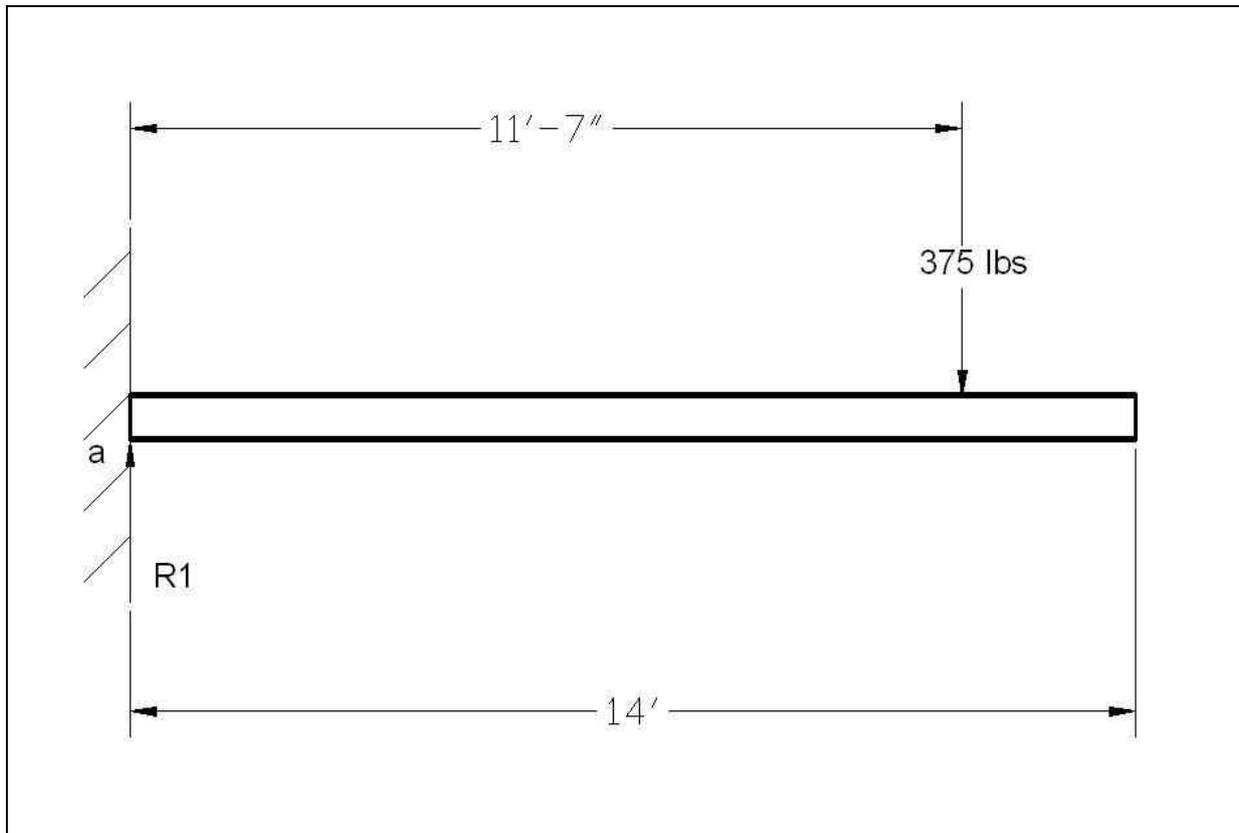


Figure 9.5 – Practice Problem for a Supported Truss with More Than One Load

## Drawing a Shear Force Diagram for a Simple Cantilever Truss

When drawing a Shear Force diagram for a cantilever truss, we follow the same steps as before, doing the free body diagram by computing the sum of the forces in the Y – direction and examining the torque in the sum of the moments.



**Figure 9.6 – Cantilever Beam with One Load**

In the first formula, we add the load, which is 375 pounds and subtract reactive force at the support R1. In the second equation, we have clockwise moments of 375 lbs × 11.583 feet.

$$\Sigma F_y = 375 \text{ lbs} - R1 = 0$$

$$\Sigma M_a = (375 \text{ lbs} \times 11.583 \text{ ft}) = 0$$

First, we solve the sum of the moment equation, by finding the foot – pounds quantity for the load. However, we see that the moment arm is 4343.625 ft – lbs clockwise. Since the beam is static, back at point A, we will draw a moment with a negative 4343.625 ft – lbs torque to counter the torque and keep the beam from twisting.

$$\Sigma M_a = (375 \text{ lbs} \times 11.583 \text{ ft}) = 0$$

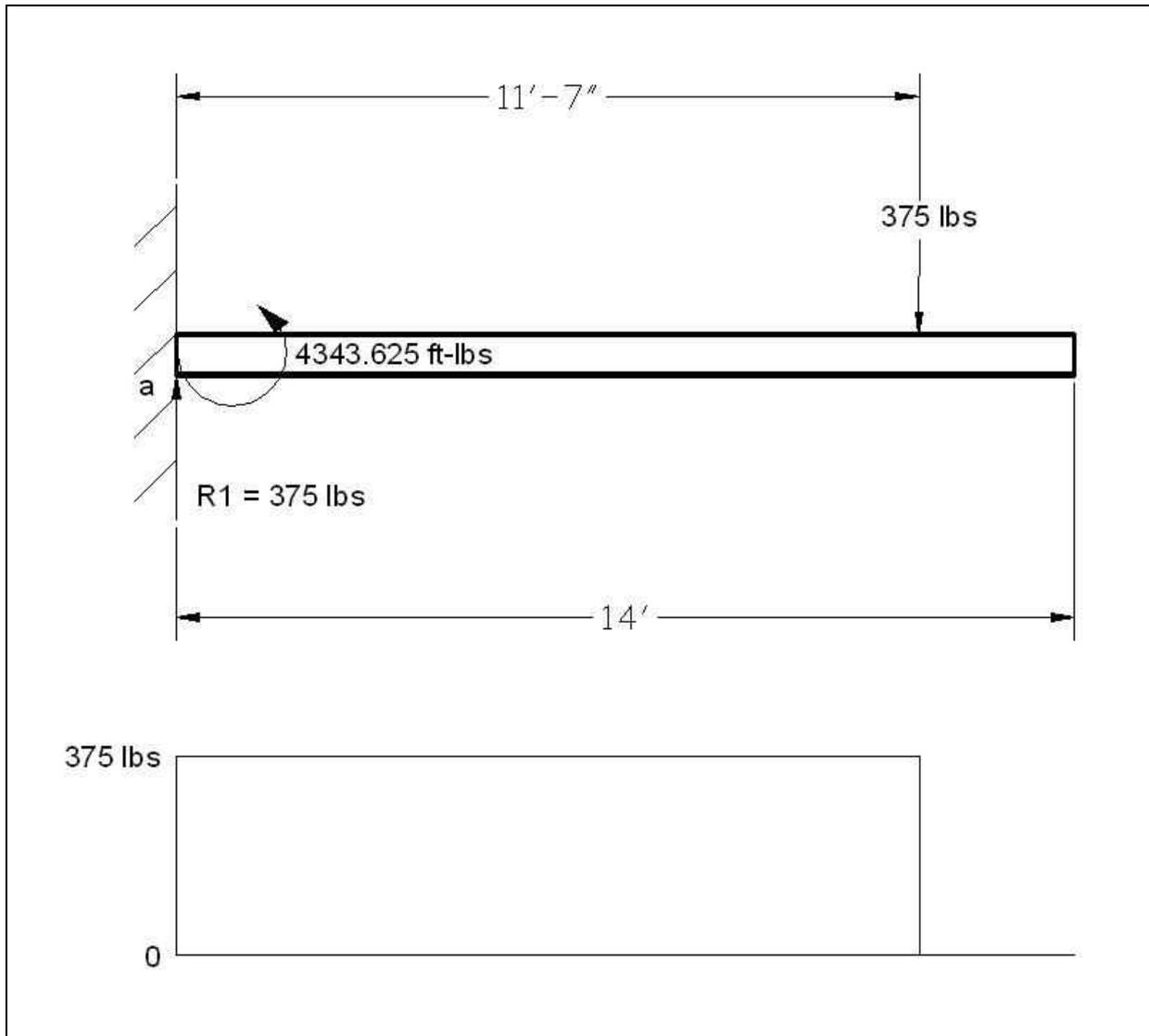
$$\Sigma M_a = 4343.625 \text{ ft -lbs}$$

After solving for sum of the moments, we utilize the sum of forces in the Y – direction to find R1. As shown below, we see the amount for R1 is 375 lbs.

$$\Sigma F_y = 375 \text{ lbs} - R1 = 0$$

$$R1 = 375 \text{ lbs}$$

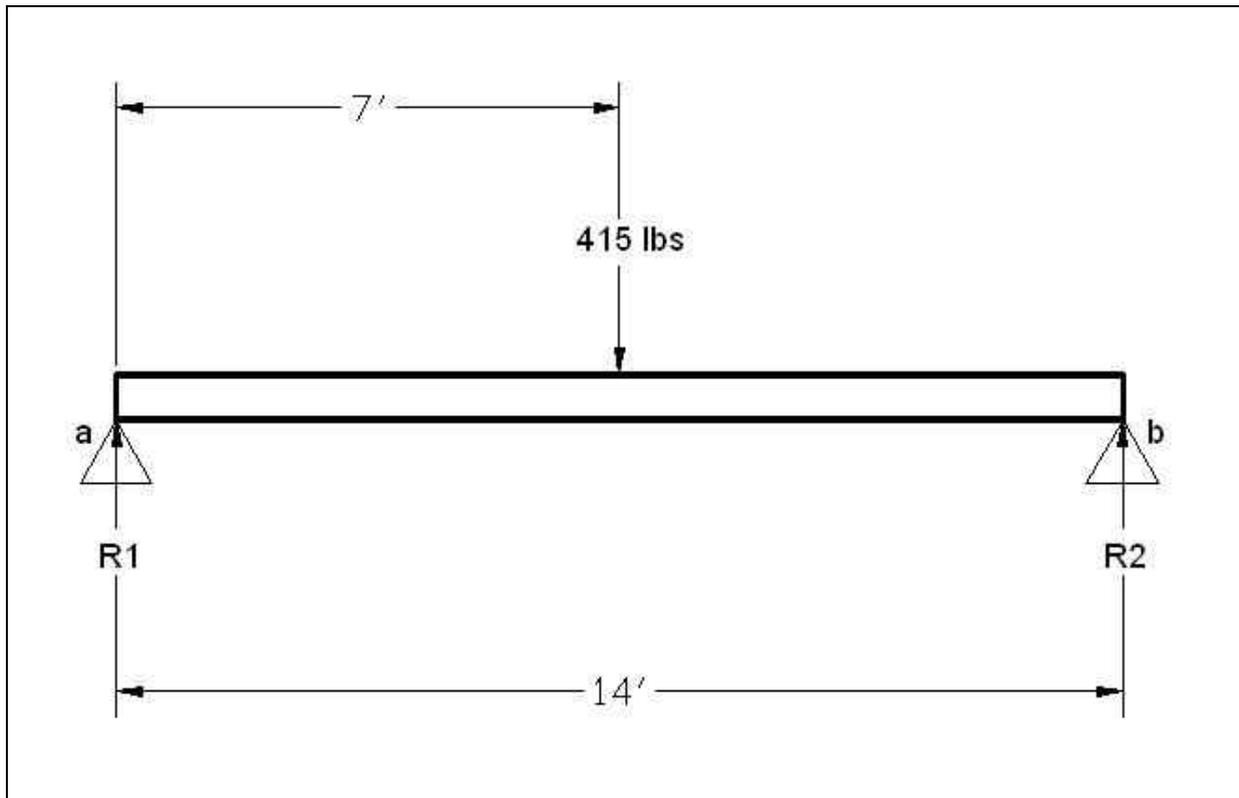
Again, we return to our CAD layout and draw the missing moment arm at point A. For the sum of the moments to equal zero, the torque at the supported end of the beam will be opposite from the rotation of the load. In this case, we draw the 4343.625 ft-lbs counterclockwise as shown in Figure 9.7.



**Figure 9.7 – Cantilever Beam with Moment Arm**

## **Drawing a Bending Moment Diagram for a Simple Truss**

To create a Bending Moment diagram, we begin with analyzing the truss with the free body diagram as we have done throughout this chapter and the last. Designers and Engineers use this graph to quickly analyze the design for bending moments. That is the force times the distance to the loads at any point of the beam.



**Figure 9.8 – Simple Truss Diagram**

When we solve a simple supported truss, we will examine the problem in equilibrium. In the problem in Figure 9.8, we sum the forces in the Y – direction, with the weight of 415 pounds acting in a positive direction and the two reactionary forces R1 and R2 repulsing the weight with in a negative path. With just this equation, we are unable to solve for the unknowns. We need to write a second formula, the sum of the moments where we scrutinize the forces acting around support A. Both equations are shown below:

$$\Sigma F_y = 415 \text{ lbs} - R1 - R2 = 0$$

$$\Sigma M_a = (415 \text{ lbs} \times 7 \text{ ft}) - (R2 \times 14 \text{ ft}) = 0$$

The sum of the moment around support A is the 415 lbs times 7 feet or 2905 ft – lbs. Divide the 2905 ft – lbs by 14 feet and we see the answer for R2 as 207.5 pounds.

$$R2 \times 14 \text{ ft} = 2905 \text{ ft} - \text{lbs}$$

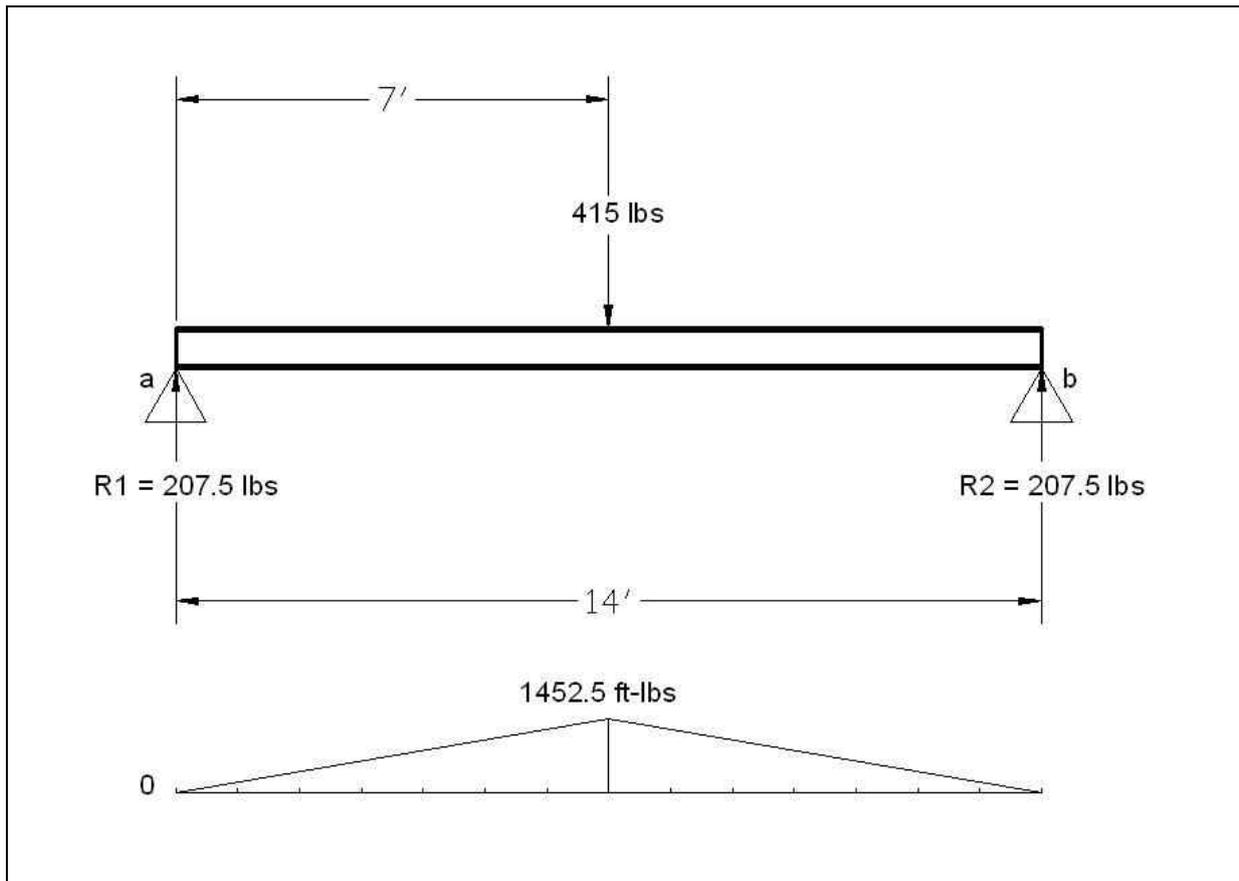
$$R2 = 2905 \text{ ft} - \text{lbs} \div 14 \text{ ft} = 207.5 \text{ lbs}$$

We use the equation for the sum of forces in the Y – direction to find R1, by plugging in the value of R2 into the formula. As shown below, we see the amount for R1 is 207.5 lbs.

$$R1 = 415 \text{ lbs} - R2$$

$$R1 = 415 \text{ lbs} - 207.5 \text{ lbs} = 207.5 \text{ lbs}$$

Now, a bending moment diagram shows the total force times the distance or moment arm at any point along the length of the truss. To compute the quantities for this diagram, we can work from left to right as we did in the Shear Force diagram. For our beam, we will start at point A. At the initial end, the distance to R1 is zero, so the moment arm is equal to zero. Next, we will examine the beam at a location one foot to the right of point A. In our work across the beam, we will examine the forces and distances to the left as we move to the right. At the one foot mark we calculate 207.5 lbs times 1 foot or 207.5 ft-lbs. We continue to move to the right one foot at a time to compute the moment arm at each incremental position.



**Figure 9.8 – Bending Moment Diagram for a Simple Truss**

Bending Moment diagrams are somewhat more difficult to draw than Shear Force graphs. A Bending Moment illustration is more likely to have sloping angles and curves, so we want to examine the data at smaller increments to get an accurate picture of what stresses are occurring in the truss. A good method to follow is to make a chart showing each calculation. There are textbooks that give solutions to a series of common applications and we can buy computer software that will do the analysis for even more difficult problems. We will make a chart to continue the assessment of the beam. We choose one foot increases across the truss, however if a force vector or uniform load starts at a point not on an even position, we have to examine that spot along the way.

Position	Moment Arm	Answer
Point A	$207.5 \text{ lbs} \times 0 \text{ ft}$	0 ft-lbs
1 foot	$207.5 \text{ lbs} \times 1 \text{ ft}$	207.5 ft-lbs
2 foot	$207.5 \text{ lbs} \times 2 \text{ ft}$	415 ft-lbs
3 foot	$207.5 \text{ lbs} \times 3 \text{ ft}$	622.5 ft-lbs
4 foot	$207.5 \text{ lbs} \times 4 \text{ ft}$	830 ft-lbs
5 foot	$207.5 \text{ lbs} \times 5 \text{ ft}$	1037.5 ft-lbs
6 foot	$207.5 \text{ lbs} \times 6 \text{ ft}$	1245 ft-lbs
7 foot	$207.5 \text{ lbs} \times 7 \text{ ft} - 415 \text{ lbs} \times 0 \text{ ft}$	1452.5 ft-lbs
8 foot	$207.5 \text{ lbs} \times 8 \text{ ft} - 415 \text{ lbs} \times 1 \text{ ft}$	1245 ft-lbs
9 foot	$207.5 \text{ lbs} \times 9 \text{ ft} - 415 \text{ lbs} \times 2 \text{ ft}$	1037.5 ft-lbs
10 foot	$207.5 \text{ lbs} \times 10 \text{ ft} - 415 \text{ lbs} \times 3 \text{ ft}$	830 ft-lbs
11 foot	$207.5 \text{ lbs} \times 11 \text{ ft} - 415 \text{ lbs} \times 4 \text{ ft}$	622.5 ft-lbs
12 foot	$207.5 \text{ lbs} \times 12 \text{ ft} - 415 \text{ lbs} \times 5 \text{ ft}$	415 ft-lbs
13 foot	$207.5 \text{ lbs} \times 13 \text{ ft} - 415 \text{ lbs} \times 6 \text{ ft}$	207.5 ft-lbs
Point B	$207.5 \text{ lbs} \times 14 \text{ ft} - 415 \text{ lbs} \times 7 \text{ ft}$	0 ft-lbs

Figure 9.9 – Computing Bending Moments for a Simple Truss

## Draw Free Body, Shear Force and Bending Moment Diagrams

Now that we know the methods for producing Free Body diagrams and we have the expertise to layout the Shear Force and Bending Moment graphs, we can create a drawing file and do all three together. Analyze the data given in figure 9.10 to find the points of interest in this truss.

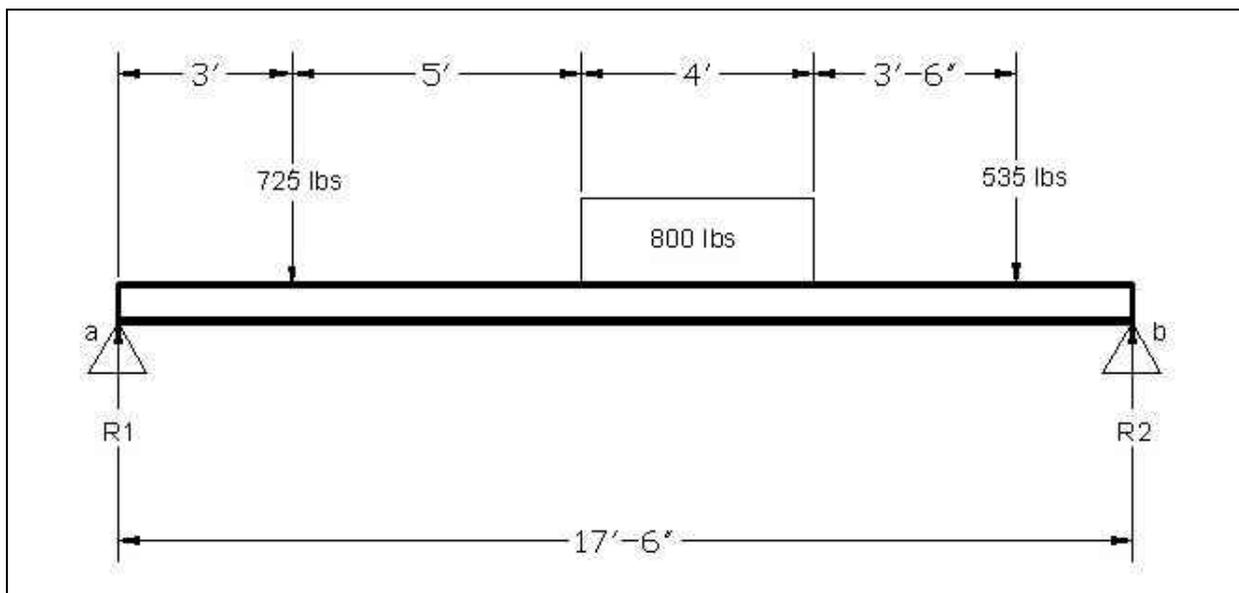


Figure 9.10 – Draw All Three Diagrams; Free Body, Shear Force and Bending Moment

**\* World Class CAD Challenge 10-15 \* - Draw a Free Body, Shear Force and Bending Moment Diagram of the truss in Figure 9.10 and indicate the points of interest in 30 minutes. Save the file as Beam Analysis.dwg.**

**Continue this drill four times using some beams problems you have determined, each time completing the drawing file in under 30 minutes to maintain your World Class ranking.**