Chapter

8

Simple Truss

In this chapter, you will learn the following to World Class standards:

- **The Simple Supported Truss**
- **The Simple Supported Truss with Two or More Loads**
- **The Simple Supported Truss with a Cantilever End**
- **The Simple Cantilever Truss**
- **Solving the Torque Problem in a Cantilever Beam**

The Simple Supported Truss

In almost every assembly, there are components that support the weight of sub assemblies within the design, so architects, designers and engineers need to be proficient in analyzing the mechanical structures that do this work. Whenever we drive by a construction site where the foundation, two by four studs, and floor and roof joists are visible, we see these mechanism that keep our household goods and ourselves safely on whatever floor level we are living. In an automobile, we would probably need to visit a factory or a repair shop to get a good look at the actual frame of the car without the exterior panels. In an electrical device like a microwave oven, the major supports will most likely bear food that we cook or just hold the exterior panels in place. No matter what the design, we need to create a specification that lists the maximum acceptable weight and a design safety factor, so we can calculate the resultant force vectors and establish the type and size of material needed.

A simple supported truss as shown in figure 8.1 has a single load representing the maximum allowable weight with a vector at each end reacting to the mass. In many cases throughout this chapter and textbook, we will ignore the weight of the beam, however always read the scenario which will inform us whether we need to apply the weight of the beam. In some cases, with our first exercises, we will solve the support problem in two dimensions, but as we move ahead in our work, we can solve problems in three dimensions.

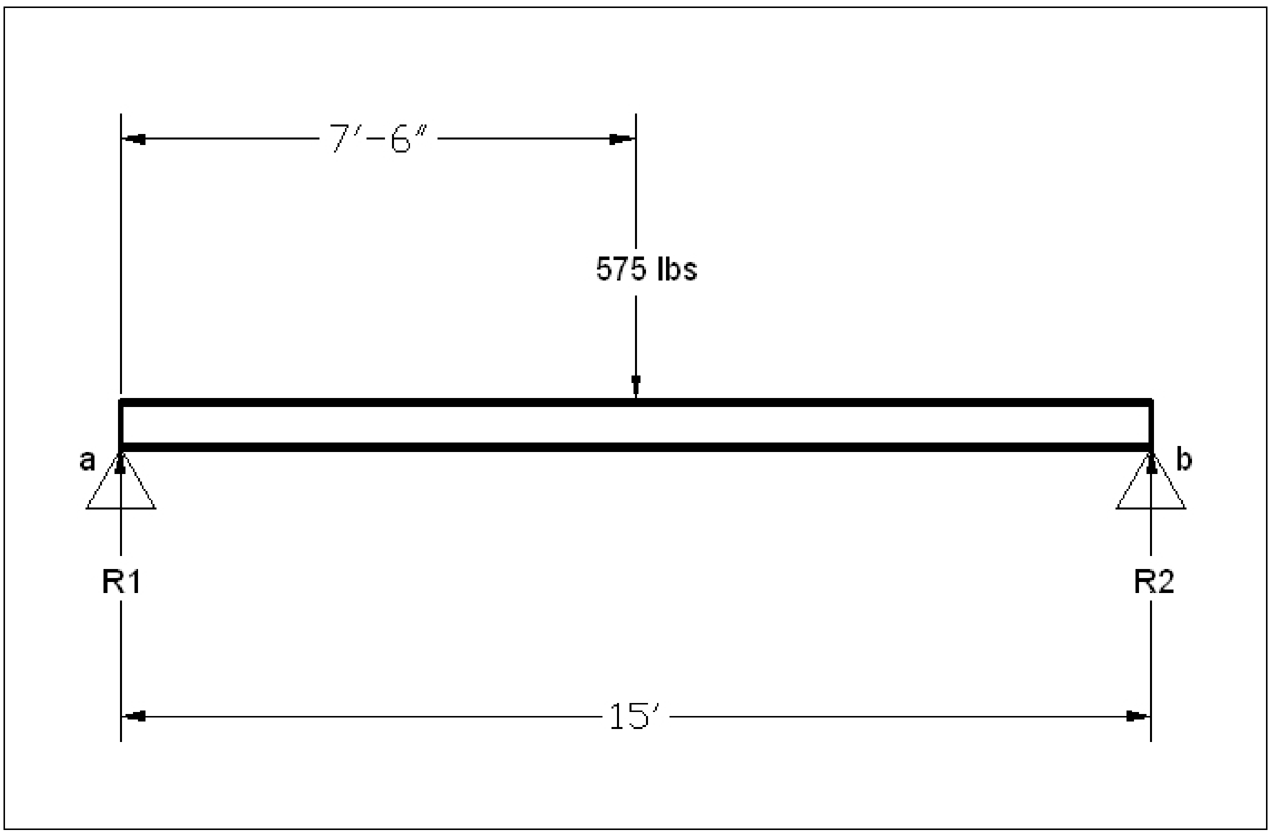


Figure 8.1 – Simple Truss

When we solve a simple supported truss, we will examine the problem in equilibrium. In the problem in figure 8.1, we add the forces in the Y – direction, with the weight of 575 pounds acting in a positive path and the two reactionary forces R1 and R2 repulsing the weight with in a negative path. With just this equation, we are unable to solve for the unknowns. We need to write a second formula, the sum of the moments where we scrutinize the forces acting around support A. Both equations are shown below:

$$\Sigma F_y = 575 \text{ lbs} - R1 - R2 = 0$$

$$\Sigma M_a = (575 \text{ lbs} \times 7.5 \text{ ft}) - (R2 \times 15 \text{ ft}) = 0$$

The sum of the moment around support A is the 575 pounds times 7.5 feet or 4312.5 ft – lbs. Divide the 4312.5 ft – lbs by 15 feet and we see the answer for R2 as 287.5 pounds.

$$R2 \times 15 \text{ ft} = 4312.5 \text{ ft} - \text{lbs}$$

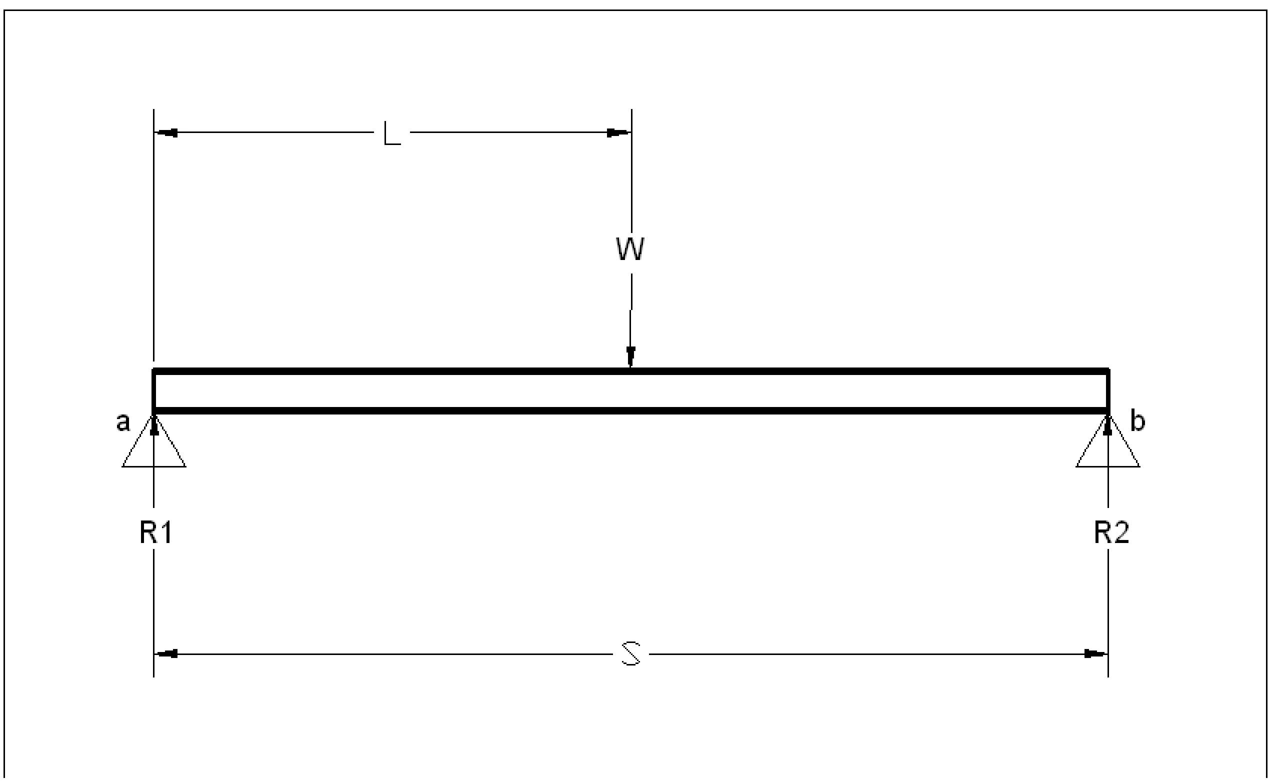
$$R2 = 4312.5 \text{ ft} - \text{lbs} \div 15 \text{ ft} = 287.5 \text{ lbs}$$

We use the equation for the sum of forces in the Y – direction to find R1, by plugging in the value of R2 into the formula. As shown below, we see the amount for R1 is 287.5 lbs.

$$R1 = 575 \text{ lbs} - R2$$

$$R1 = 575 \text{ lbs} - 287.5 \text{ lbs} = 287.5 \text{ lbs}$$

If the force is not perfectly centered, we still can calculate the reactive forces in this type of problem. Solve the next ten simple supported beam problems using data from the table and drawing in figure 8.2.



	Span (S)	Length (L)	Weight (W)	R1	R2
1	15'	10'	1000		
2	15'	5'	1100		
3	20'	10'-6"	1400		
4	20'	16'-6"	1450		
5	25'	20'-4"	1500		
6	26'-6"	17'-6"	2000		
7	28'-6"	13'-8"	2100		
8	29'	15'-6"	1500		
9	30'	25'	2000		
10	30'	2'	1500		

Figure 8.2 – Simple Truss Practice Problems

The Simple Supported Truss with One or More Loads

In some assemblies, there are two or more loads on the supported truss, such as the three loads shown in figure 8.3. In this case, the center of each load is displayed on the beam with a measurement referencing from one end. We will still apply the sum of forces in the Y – direction and the sum of the moments around point A. Multiple loads create more calculations in the formulas, but once we have the method well practiced; we can unravel these problems as well.

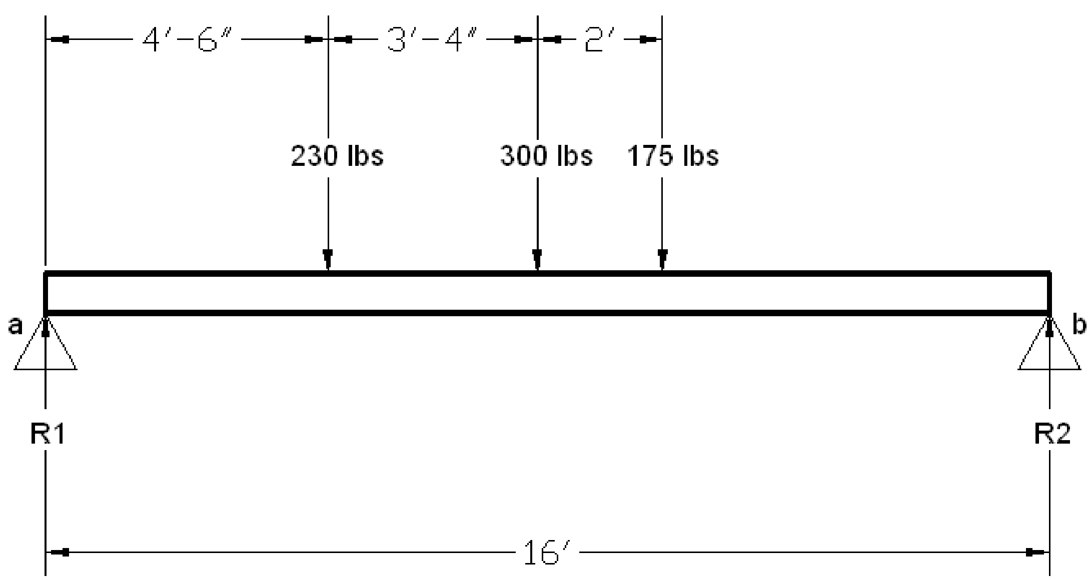


Figure 8.3 – Simple Supported Truss with More Than One Load

In the first formula, we add the three loads which are 230 pounds, 300 pounds and 175 pounds and subtract reactive forces at the supports R1 and R2. In the second equation, we have clockwise moments of 230 lbs × 4.5 ft, 300 lbs × 7.833 ft, and 175 lbs × 9.833 ft. The last moment is in a counterclockwise direction and is R2 × 16 ft.

ΣFy = 230 lbs + 300 lbs + 175 lbs – R1 – R2 = 0

ΣMa = (230 lbs × 4.5 ft) + (300 lbs × 7.833 ft) + (175 lbs × 9.833 ft) – (R2 × 16 ft) = 0

First, we solve the sum of the moment equation, by finding the foot – pounds quantity for each load and then divide by 16 feet to obtain force R2. The value for R2 is 340.378 lbs as shown below.

ΣMa = (230 lbs × 4.5 ft) + (300 lbs × 7.833 ft) + (175 lbs × 9.833 ft) – (R2 × 16 ft) = 0

R2 = 1035 ft. – lbs + 2349.9 ft. – lbs + 1720.775 ft. – lbs ÷ 16 ft.

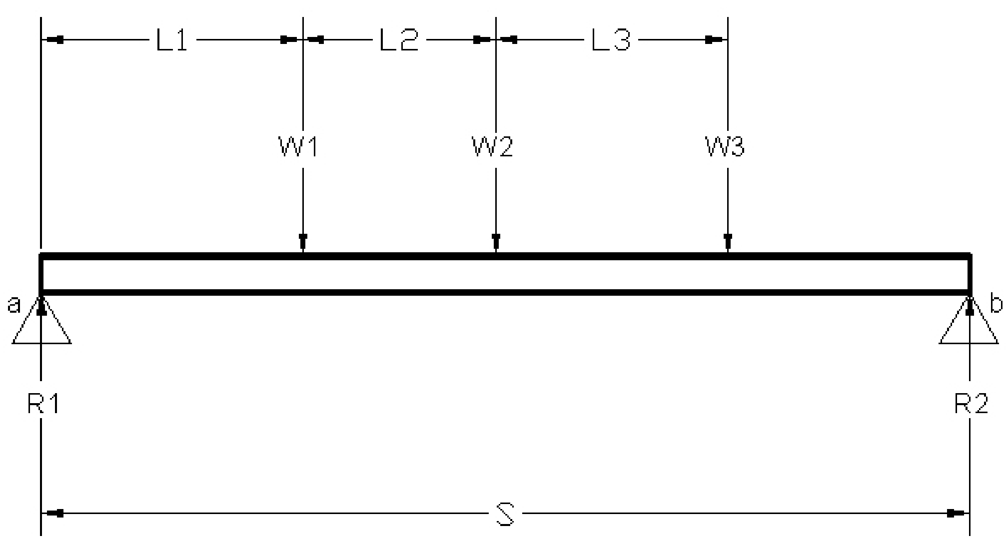
R2 = 5105.675 ft. – lbs ÷ 16 ft. = 319.1046875 lbs.

After solving for force R2, we utilize the sum of forces in the Y – direction to find R1, by plugging in the value of R2 into the formula. As shown below, we see the amount for R1 is 385.8953125 lbs.

ΣFy = 230 lbs + 300 lbs + 175 lbs – R1 – R2 = 0

R1 = 705 lbs – 319.1046875 lbs = 385.8953125 lbs

Solve the next ten multiple loaded beam problems using data from the table and drawing in figure 8.4.



	Span (S)	Length (L1)	Weight (W1)	Length (L2)	Weight (W2)	Length (L3)	Weight (W3)	R1	R2
1	16'-0"	1'-6"	200	4'-0"	250	2'-6"	175		
2	18'-0"	3'-0"	300	5'-6"	300	3'-0"	150		
3	20'-0"	2'-6"	100	1'-8"	214	1'-9"	320		
4	22'-6"	2'-3"	240	1'-6"	122	2'-3"	295		
5	16'-0"	8'-0"	120	3'-6"	410	4'-6"	162		
6	21'-0"	3'-2"	310	3'-0"	352	1'-8"	231		
7	19'-10"	4'-1"	182	1'-10"	325	5'-8"	373		
8	20'-1"	3'-0"	215	2'-3"	237	1'-6"	512		
9	25'-0"	3'-7"	320	7'-6"	125	--	--		
10	40'-0"	5'-2"	400	9'-6"	523	--	--		

Figure 8.4 – Practice Problem for a Supported Truss with More Than One Load

The Simple Supported Truss with a Cantilever End

When there is a roof truss overhanging the edge of the wall of a house, the one end of the truss is unsupported which is called a cantilever beam. In figure 8.5, we have a 500-pound force on the end of the joist. Using the techniques we have learned, we will solve for the unknown forces R1 and R2.

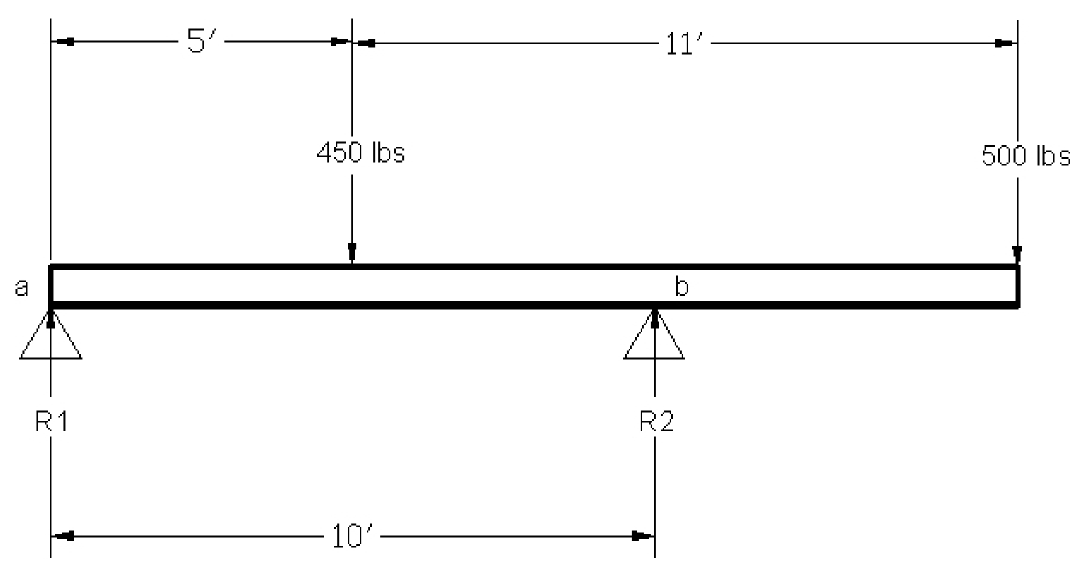


Figure 8.5 – Simple Supported Beam with a Cantilever Load

In the first formula, we add the two loads, which are 450 pounds and 500 pounds and subtract reactive forces at the supports R1 and R2. In the second equation, we have clockwise moments of 450 lbs × 5 ft and 500 lbs × 16 ft. The last moment is in a counterclockwise direction and is R2 × 10 ft.

ΣFy = 450 lbs + 500 lbs – R1 – R2 = 0

ΣMa = (450 lbs × 5 ft) – (R2 × 10 ft) + (500 lbs × 16 ft) = 0

First, we solve the sum of the moment equation, by finding the foot – pounds quantity for each load and then divide by 15 feet to force R2. The value for R2 is 1025 lbs as shown below.

ΣMa = (450 lbs × 5 ft) – (R2 × 10 ft) + (500 lbs × 16 ft) = 0

R2 = 2250 ft. – lbs + 8000 ft. – lbs ÷ 10 ft.

R2 = 10250 ft. – lbs ÷ 10 ft. = 1025 lbs.

After solving for force R2, we utilize the sum of forces in the Y – direction to find R1, by plugging in the value of R2 into the formula. As shown below, we see the amount for R1 is -75 lbs.

ΣFy = 450 lbs + 500 lbs – R1 – R2 = 0

R1 = 950 lbs – 1025 lbs = -75 lbs

With the force vector R1 being a negative value, the initial beam layout drawn in figure 8.5 is not entirely correct. The actual description of the forces acting on the joist is shown in figure 8.6 where R1 is now pointing downward similar to the 450 and 500-pound loads. It is not necessary to redraw the diagram, but when we are beginning to learn, we will draw the sketch accurately.

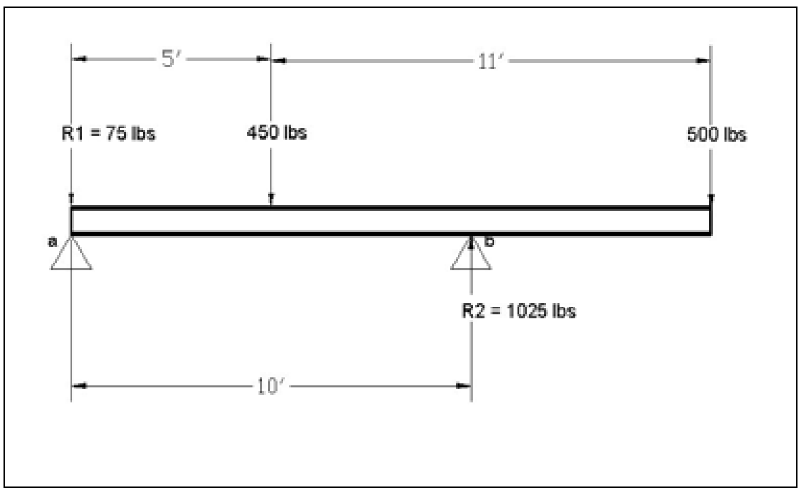


Figure 8.6 – Corrected Beam Layout

The Simple Cantilever Truss

The simple cantilever truss is very common in engineering. When traffic lights hang off a horizontal pole projected from vertical structure, the hanging joist is cantilever. When a sign is suspended from a steel bar projecting from the wall of a building, the metal bar is cantilever. We use the same formula to analyze the beam, solving for the forces in the Y –direction and the sum of the moments from the attached end, point A.

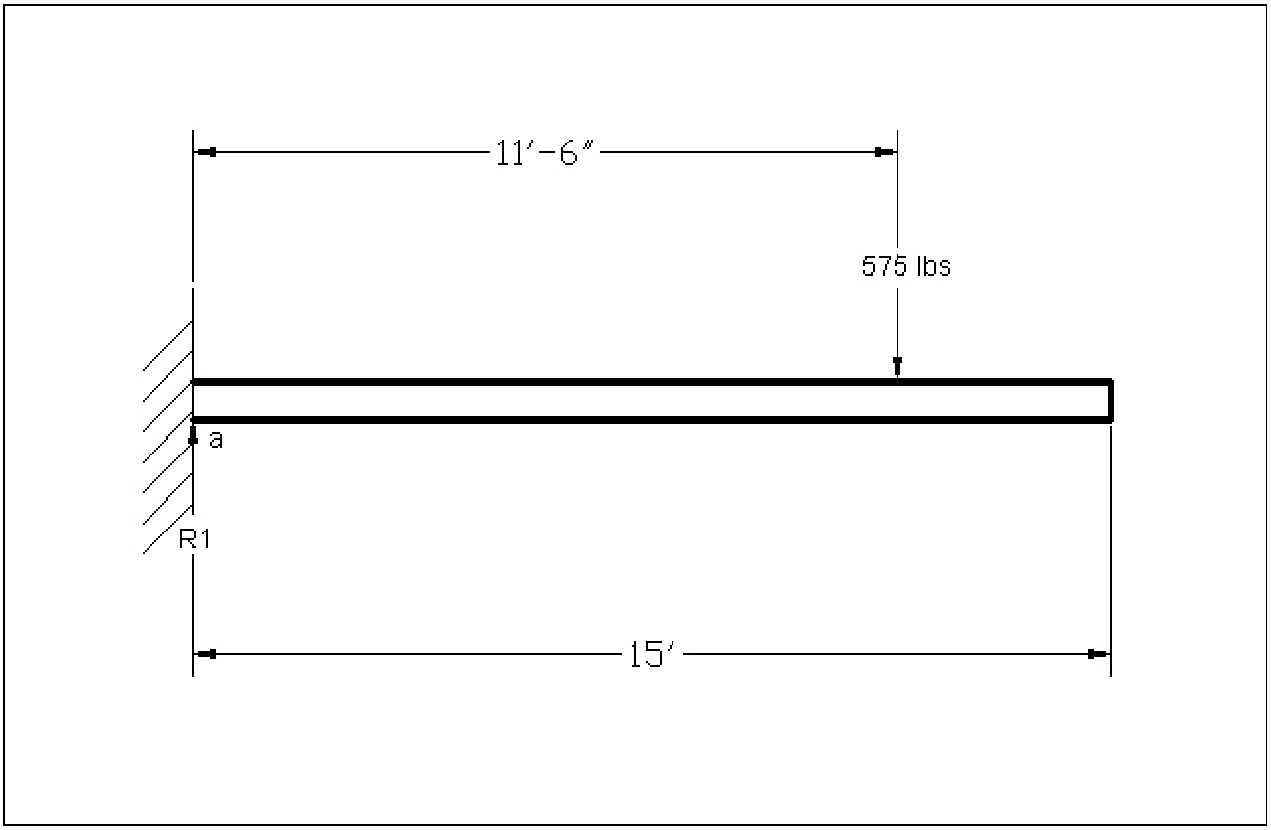


Figure 8.7 – Cantilever Beam with One Load

In the first formula, we add the load, which is 575 pounds and subtract reactive force at the support R1. In the second equation, we have clockwise moments of 575 lbs × 11.5 feet.

ΣFy = 575 lbs – R1 = 0

ΣMa = (575 lbs × 11.5 ft) = 0

First, we solve the sum of the moment equation, by finding the foot – pounds quantity for the load. However, we see that the moment arm is 6612.5 ft – lbs clockwise. Since the beam is static, back at point A, we will draw a moment with a counterclockwise 6612.5 ft – lbs torque to resist the torque and keep the beam from twisting.

ΣMa = (575 lbs × 10.5 ft)   = 0

ΣMa = 6612.5 ft -lbs

After solving for sum of the moments, we utilize the sum of forces in the Y – direction to find R1. As shown below, we see the amount for R1 is 575 lbs.

ΣFy = 575 lbs – R1 = 0

R1 = 575 lbs

Again, we return to our CAD layout and draw the missing moment arm at point A. For the sum of the moments to equal zero, the torque at the supported end of the beam will be opposite from the rotation of the load. In this case, we draw the 6612.5 ft-lbs counterclockwise as shown in figure 8.8.

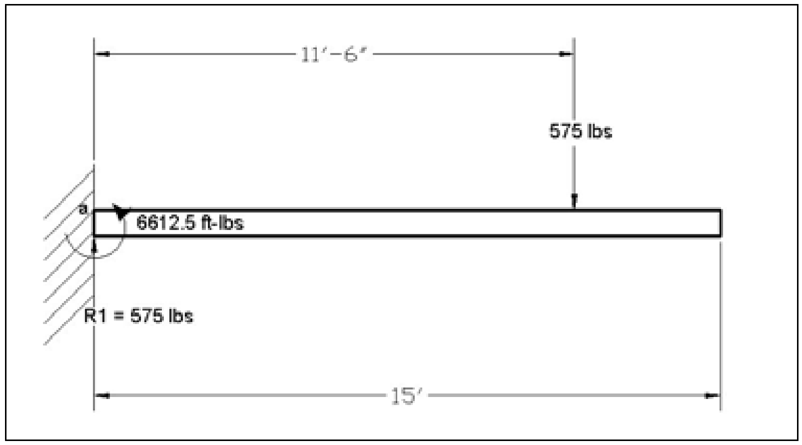


Figure 8.8 – Cantilever Beam with Moment Arm

Solving the Torque Problem in a Cantilever Beam

To remove the 6612.5 foot – pound torque from the supported end of the cantilever beam, we typically place a cable from the unsupported end to the wall. This will introduce a resultant vector working in both the X and Y direction. We will solve for the unknowns using equations for the sum of forces in the X and Y paths and for the sum of the moments around point A.

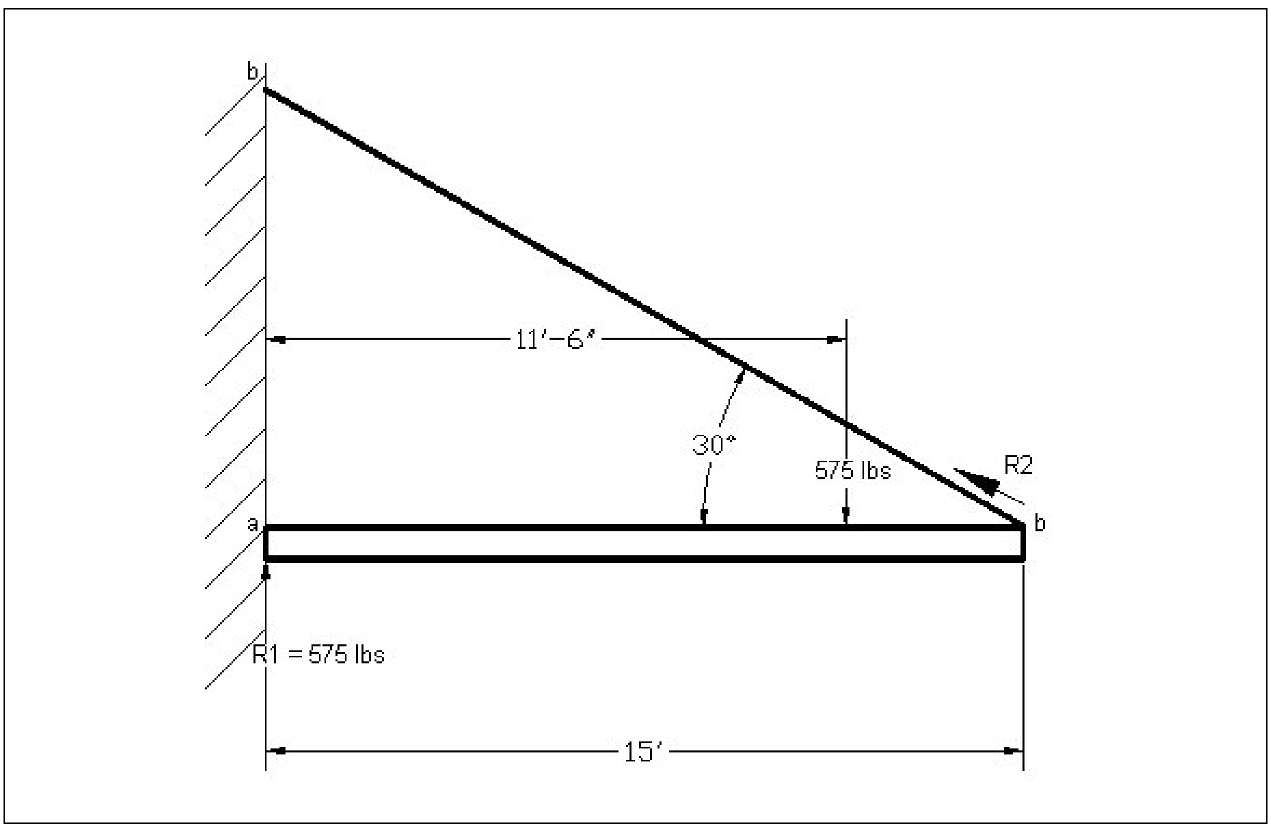


Figure 8.9 – Adding a Cable to the End of the Cantilever Beam

In the first formula, we add the load, which is 575 pounds and subtract reactive force at the support R1. In the second equation, we have clockwise moments of 575 lbs × 11.5 feet.

ΣFy = 575 lbs – R1y – R2y = 0

ΣFx = R1x – R2x = 0

We use our CAD layout to find the distance from point A to the cable as shown in figure 8.10 and place the measurement in the sum of the moment formula.

ΣMa = (575 lbs × 11.5 ft) – (R2 × 7.5 ft) = 0

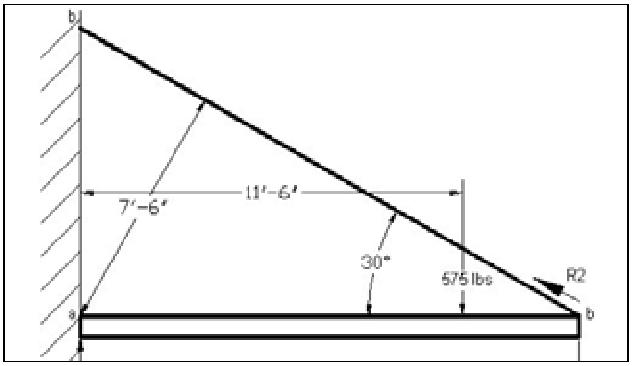


Figure 8.10 – 7'6" to Force R2

First, we solve the sum of the moment equation, by finding the foot – pounds quantity for the load. However, we see that the moment arm is 6612.5 ft – lbs clockwise. Since the beam is static, back at point A, we will draw a moment with a negative 6612.5 ft – lbs torque to counter the torque and keep the beam from twisting.

ΣMa = (575 lbs × 11.5 ft) – (R2 × 7.5 ft) = 0

R2 = 6612.5 ft –lbs ÷ 7.6 ft = 870.1 lbs

After solving for sum of the moments, we break down the resultant vector R2 into the subcomponents R2y and R2x. The physical position of 30° degrees proportions the force in R2 into R2 × 0.500 or 435.0 lbs for R2y. The force of R2x is R2 × 0.866 or 753.5 lbs. Using CAD to solve for sub members of the resultant force is way to find the answer. Some professionals will use the sine of 30° time R2 to get 435.0 lbs and the cosine of 30° times R2 to get 753.5 lbs.

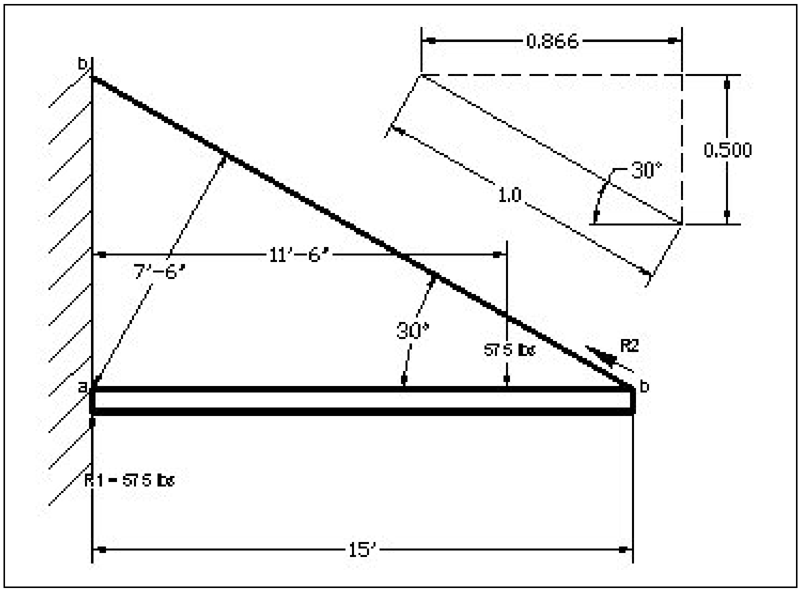


Figure 8.11 – 7'6" to Force R2

After solving for the subcomponents of R2, we utilize the sum of forces in the X – direction to find R1, by plugging in the value of R2x into the formula. As shown below, we see the amount for R1x is 753.5 lbs.

ΣFx = R1x – R2x = 0

R1x = 753.5 lbs

The forces in the Y direction are 575 lbs downward, and 435 lbs and R2y upwards. This gives us 140 lbs at R1y

ΣFy = 575 lbs – 435 lbs – R1y = 0

R1y = 575 lbs – 435 lbs = 140 lbs

The resultant vector at point A is R1x of 753.5 lbs and R1y of 140 lbs. To find the resultant vector that consists of magnitude and direction, we add the two subcomponents as we did in chapter 3 and shown in figure 8.12.

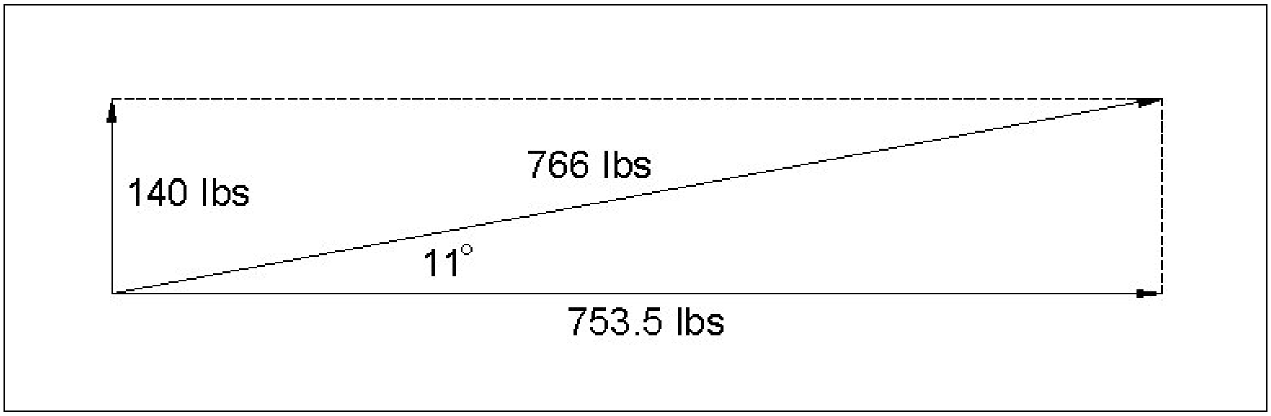


Figure 8.12 – The Resultant Force for R1 at Point A

The resultant force R1 is 766 lbs. at 11°.

In every truss problem in the chapter, we use the sum of the forces and the sum of the moments to solve for unknown values. We can practice these methods on any type of structure we see. In the next chapter, we will continue to study beams and joists to analyze their weak spot and give as information so we can select the right beam for the job.