Chapter 7

Equilibrium in Three Dimensions

In this chapter, you will learn the following to World Class standards:

- **Forces in Three Different Axis**
- **Wind Load on the Antennae Pole**
- **Practice Problem - Compute Wind Load and Forces on a Pole**

Forces on an Antennae Pole

In this chapter, we will study a series of projects that have force vectors interacting in more than one plane, so therefore we have to develop techniques which break down the resultant vectors into subcomponents in the X, Y and Z axis. In these pages, we will continue to incorporate previous lessons in mechanics and mechanical design that rely upon knowledge of mass, density and developing basic concepts of how assemblies interact with physical laws on this planet.

We made a 34-foot tall steel pole with a one-inch thick wall at the 12-inch diameter base in our Computer Aided Design (CAD) program. The pole tapers on the outside at 0.5° and on the inside diameter at 0.4°, so the thickness of the metal at the top is approximately 5/16 inch thick. The pole was set in a concrete foundation but did not have any tethers holding the structure. We want to add three stainless steel cables equally spaced around vertical structure 20 feet out and projecting up 45°. There is a tension of 500 pounds on each of the cables.

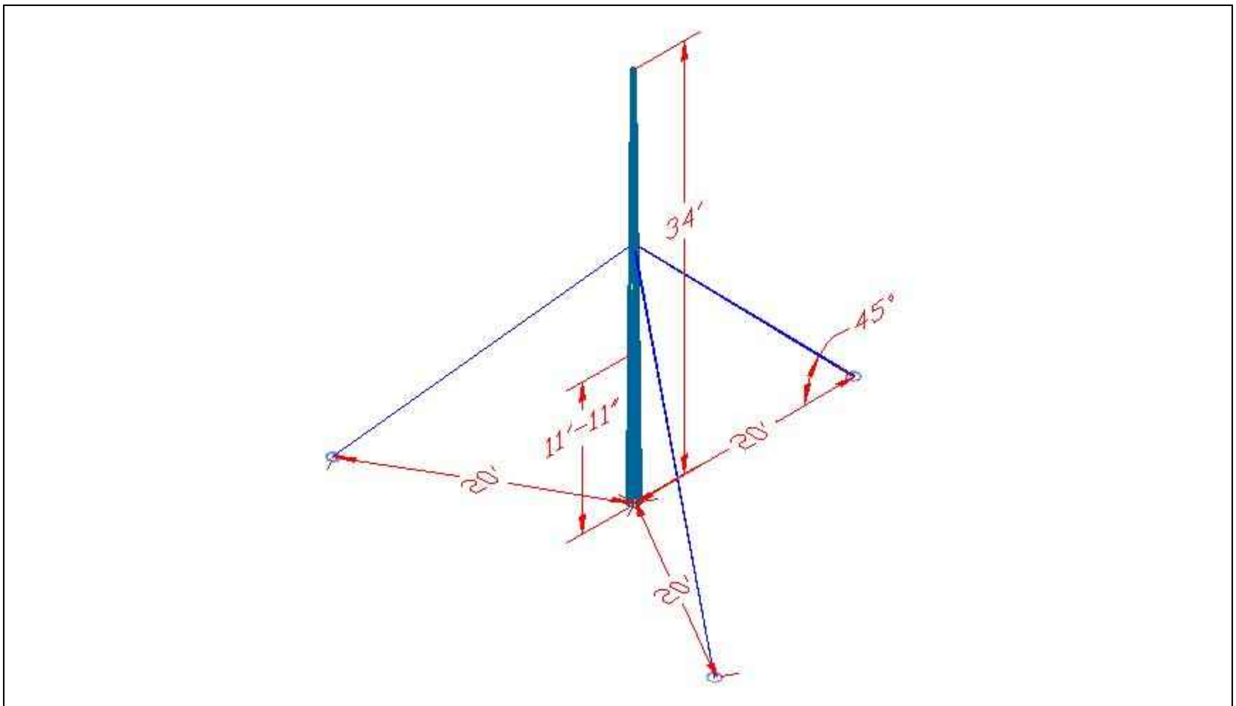


Figure 7.1 – The Antennae Pole

In figure 7.2, we see the force breakdown of each cable. Remember, we added several vectors to obtain a resultant. Now, we take the tension in each guy cable and find the X, Y and Z subcomponents of the main force. For example, we display cable F1 as F1x and F1z. There is no force in the Y direction, so we omit the F1y. F1x is pointing in the positive X direction and F2x and F3x are shown facing 180° in the opposite bearing. We also see that vector F2 equals vector F2x plus vector F2y plus vector F2z. In figure 7.3, we see all of the forces acting on the antennae pole on a calm day without wind pushing on the side which will affect the reaction of force F4 at the base.

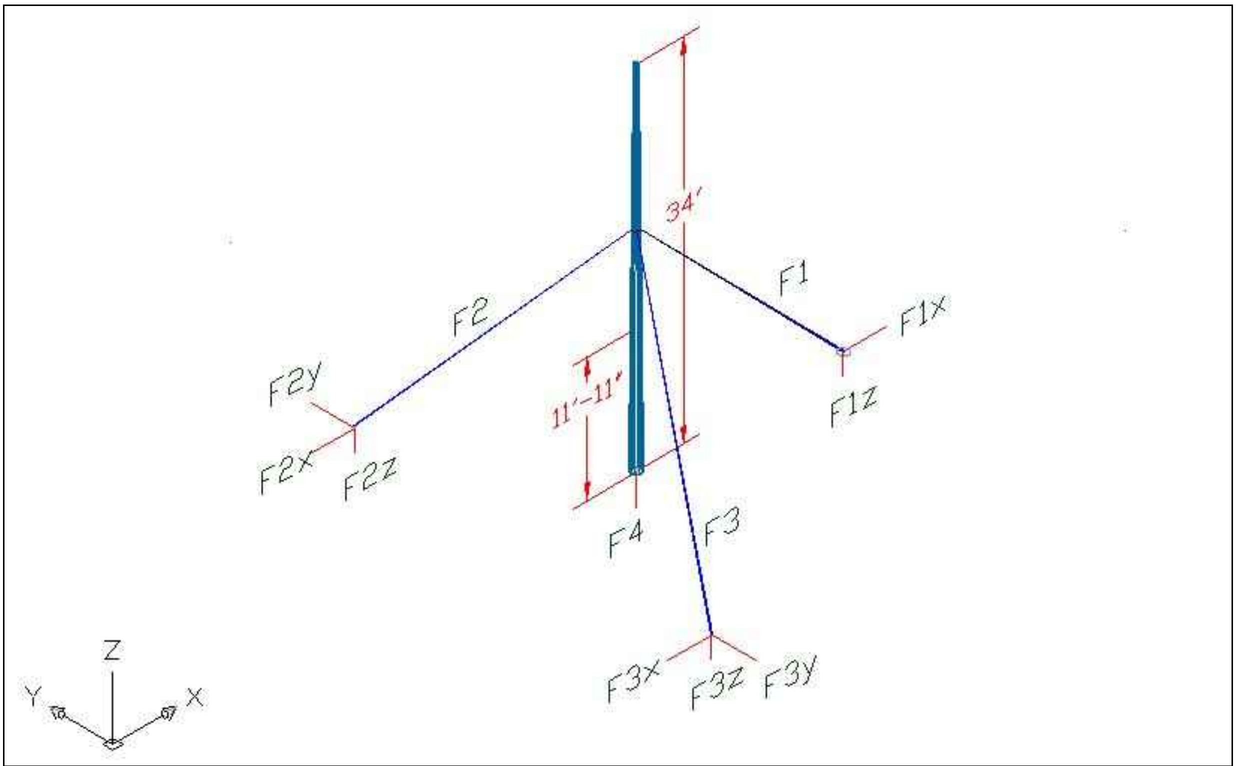


Figure 7.2 – The Antennae Pole

As we can already see, the foundation of the pole has to withstand the weight of the metal tube, the total weight of the three sets of guy wires and brackets and the tension of the cables.

Item	Force
Steel Pole	1961.54 lbs
3 – 0.50 in dia. Cables and brackets	75.00 lbs
Force Z of the Guy Cabels	Unknown
Total	Unknown

Figure 7.3 – Forces on the Foundation Base

Anyone can find the weight of the steel pole by taking the volume of the 3D solid (6931.2378 in³) and multiply by the density of steel (0.283 lb / in³). The weight of the pole as shown in figure 7.3 is 1961.54 lbs.

We can solve for the unknown force in the Z-direction using the sum of the forces. In the sketch we have drawn in our CAD program, we can see the positive direction of the X, Y and Z axis by noticing the direction of the arrows in the coordinate icon in the lower left corner of figure 7.2.. If a subcomponent force is pointing in the positive direction then the value will have a plus sign in our equation. If a sub element force is facing in the opposite bearing then the amount shown will have a negative sign. In the static problem all sums of forces will equal zero.

ΣFx = F1x – F2x – F3x = 0
ΣFy = F2y – F3y = 0
ΣFz = F4 – F1z – F2z – F3z – Wt (pole) – Wt (guy lines) = 0

We already know that F1 equals F2 and F3, but as we expand on this problem in the next section, we will take the time to analyze some data and prove to ourselves mathematically the values of the sub forces. We will rewrite the above formulas to examine relationships.

F1x = F2x + F3x
F2y = F3y
F4 = F1z + F2z + F3z + Wt (pole) + Wt (guy lines)

F1x = F2x + F3x and since F2x and F3x have equal vector lengths, ½ F1x equals F2x or ½ F1x equals F3x. Now, we will solve for the subcomponent forces in the first guy line, F1.

To compute the subcomponent forces of F1, we examine our CAD drawing and discover that both legs of the 45° vector are 0.7071 of the resultant. That is the same value as the sine or cosine of 45°. So, we calculate:

F1x = 0.7071 (500 lbs)
F1x = 353.55 lbs

F1z = 0.7071 (500 lbs)
F1z = 353.55 lbs

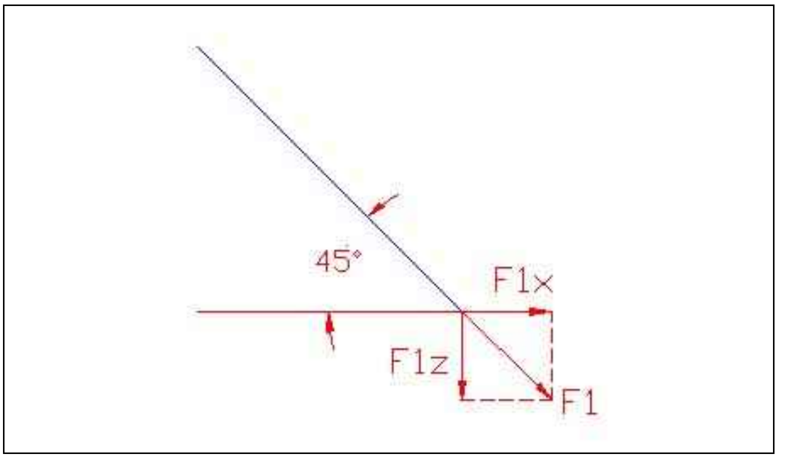


Figure 7.4 – Subcomponent Forces of F1

Since we know that ½F1x equals F2x, then F2x calculates as 0.5 times 353.55 lbs or 176.775 lbs. Now, we will use the length of F2x to figure the measurement of the other two subcomponent vectors.

As we see in figure 7.5, we encompass the actual guy line in a box (shown with hidden lines) to determine the length of each X, Y and Z leg in relationship to the true distance. As we saw this relationship in the two dimensional layout in figure 7.4, we add the third axis to account for any force. Where the two axis technique works for forces operating in one plane, the diagram in figure 7.5 works for any force, so we can solve a wider range of design problems. Now, we will calculate values to solve the unknowns.

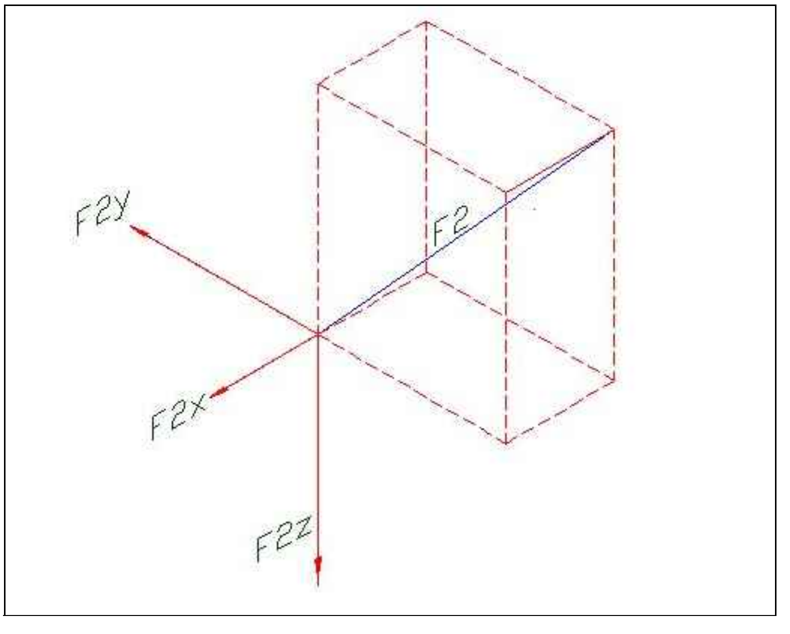


Figure 7.5 – Subcomponent Forces of F2

In our drawing, we measure a straight segment of the cable as 339.4113 inches. Then we list the information for the line segment we have drawn showing the lengths of Delta X, Delta Y and Delta Z that are given to us by the CAD program. After dividing the length of the smaller axis members by the true length of the resultant we are able to obtain the proportional values of the force vector F2 that are shown in the last column in figure 7.6.

Axis	Length (inches)	Proportion of F2
X	120	0.3536 F2
Y	240	0.7071 F2
Z	207.8461	0.6124 F2

Figure 7.6 – Proportional Values of the Subcomponent Forces of F2

So since we know that F2 is 500 lbs, we can find the values of all three vectors.

0.3536×500 lbs. = 176.8 lbs in the X axis
0.7071×500 lbs. = 353.55 lbs in the Y axis
0.6124×500 lbs. = 306.2 lbs in the Z axis

Previously, we calculated that F2x as 176.775 lbs using the force vectors in one plane and we see that computing the sub vectors of the cable moving in three paths, we have a similar answer using the three dimensional method.

Finally, to calculate the resultant force F4, we use the 306.2 pound force in the Z direction to complete the formula:

F4 = F1z + F2z + F3z + Wt (pole) + Wt (guy lines)

F4 = 306.2 lbs + 306.2 lbs + 306.2 lbs + 1961.54 lbs + 75 lbs = 2955.14 lbs

Besides determining the total weight in pounds upon the foundation of the pole, we were able to review techniques that are helpful in calculating the subcomponents of a resultant force by examining the physical characteristics of the assembly. In our next problem, we will add a force to the side of the vertical structure, which we will evaluate by determining the amount of force that wind loads add to the problem.

Wind Load on the Antennae Pole

With the same drawing, we decide that we need to withstand a 100 mph wind force. We will examine the forces on the structure if a 100 mph constant wind exerts force on the west side of the pole as shown in figure 7.7. Then we will find the force on each guy line and at the base. To compute the wind load, we will use a formula that gives us the drag force of wind against the cylindrical surface of the pole. Then, still having the initial 500 pound tension in the support cables, we can compute all of the other unknown quantities in the problem.

As in the previous mechanic problems, we locate the Center of Gravity for the pole using the Mass Properties tool and we will place the force vector we calculate for the wind load at that point. For our structure, the Center of Gravity is 11 feet and 11 inches (11.917 ft) above the base of the pole. In figure 7.7, we show the force perpendicular to cable F1.

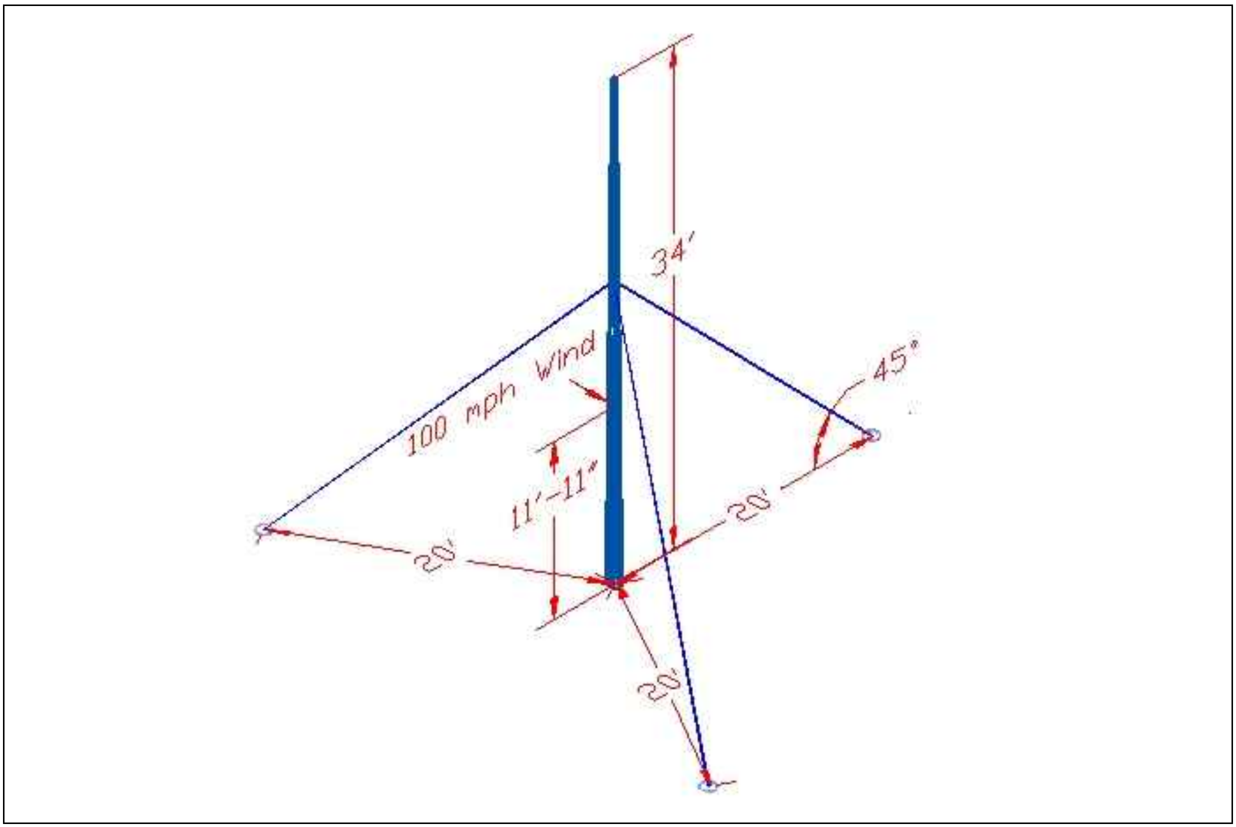


Figure 7.7 – Conducting the First Experiment

The formula for computing the force of drag on the pole is:

$$F_d = \frac{1}{2} Cd \times A \times \rho \times V^2$$

Where each variable is shown in the table in figure 7.8.

Variable	Description	Unit
F_d	Force of drag	Lbs.
Cd	Coefficient	---
A	Area (cross section)	ft^2
ρ	Air Density	$lb\text{-}sec^2/ft^4$
V	Air Velocity	ft/sec

Figure 7.8 – Variables in the Formula to Computer the Force of Drag

The first task we will perform is to change 100 mph into feet per second.

$$\frac{100 miles}{hr} \times \frac{hr}{3600 \sec} \times \frac{5280\, ft}{mile} = 146.7 \text{ ft/sec}$$

Now that we have a value of 146.7 ft /sec for the wind speed, we will use the CAD drawing to find the cross sectional area of the pole as 23.9767 square feet. We accomplished this by determining the flat area the pole has made to block the air from moving. The density of air is 0.0024 lb-sec^2/ft$^{4.}$ The coefficient of drag (Cd) for a cylindrical shape perpendicular to the direction of air flow is 1.2 as shown in figure 7.9.

$$F_d = \frac{1}{2}(1.2) \times (23.9767\, ft^2) \times 0.0024 \frac{lb - \sec^2}{ft^4} \times (146.7 \frac{ft}{\sec})^2$$

$$F_d = \frac{1}{2}(1.2) \times (23.9767\, ft^2) \times 0.0024 \frac{lb - \sec^2}{ft^4} \times 21520.89 \frac{ft^2}{\sec^2} = 743.04 \text{lbs}$$

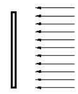
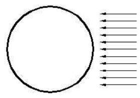
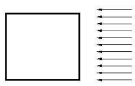
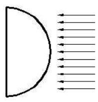
Common Shapes	**Cd**	**Image**
Flat plate	2.0	
Cylinder	1.2	
Rectangular Tube	2.0	
Hemisphere	1.3	

Figure 7.9 – Coefficient of Drag for Different Shapes

This is a very simple method to determine the force that wind places on an assembly. To calculate wind loads on a building is more complex. Now, using figure 7.11, we will calculate the forces on the pole with the added wind load, by figuring the sum of the forces in the three directions.

ΣFx = F1x – F2x – F3x = 0
ΣFy = F2y – F3y – 743.04 lbs + F4y = 0
ΣFz = F4z – F1z – F2z – F3z – Wt (pole) – Wt (guy lines) = 0

In this evaluation, we use the sum of the moments at the intersection of the guy lines which we call point G. Of course, the forces at point G where the guy lines connect to the pole will be zero because the distance is zero. The two forces we examine are the wind load at 743.04 pounds times 8.083 feet and in the other direction, F4y times 20 feet. We rearrange the equation to solve for F4y.

$$\Sigma Mg = 743.04 \text{ lbs } (8.083 \text{ ft}) - F4y \, (20 \text{ ft}) = 0$$

$$F4y = 743.04 \text{ lbs } (8.083 \text{ ft}) \div 20 \text{ ft} = 300.30 \text{ lbs}$$

So we can plug the F4y data of 300.30 pounds into the ΣFy equation to get:

$$\Sigma Fy = F2y - F3y - 743.04 \text{ lbs} + 300.30 \text{ lbs} = 0$$
$$F2y = F3y + 442.74 \text{lbs}$$

We still use the proportional data from the physical shape of the structure to determine the value of the sub forces. These forces are shown in the table in figure 7.10 and will help us compute the forces.

Axis	Proportion of F1	Proportion of F2	Proportion of F3
X	0.7071 F1	0.3536 F2	0.3536 F3
Y	---	0.7071 F2	0.7071 F3
Z	0.7071 F1	0.6124 F2	0.6124 F3

Figure 7.10 – Proportional Forces as Found Using Delta X, Y and Z of the Cable

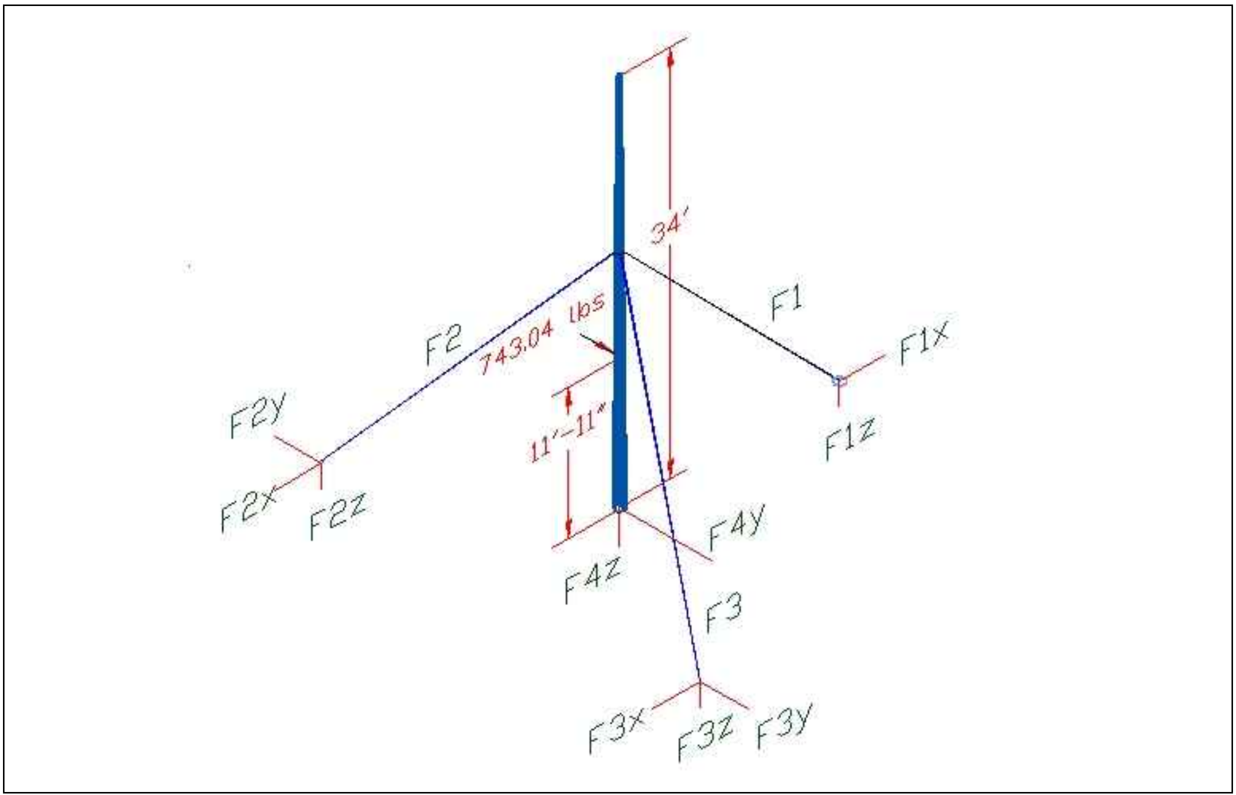


Figure 7.11 – Sketch of the Antennae Pole with Sub Vectors

We measure the force on cable F3 and find the tension to be 500 pounds, but the cable F2 has to resist the load and will increase in stress. If the tension in F3 is 500 pounds, then F3y is 353.55 pounds and F3x is 176.8 pounds.

F2y = F3y + 442.74lbs
F2y = 353.55 lbs + 442.74 lbs = 796.29 lbs

Now, since F2y equals 0.7071 F2, then F2 equals F2y ÷ 0.7071. The resultant force of cable F2 is 796.29 lbs ÷ 0.7071 = 1126.135 lbs.

To calculate F2x, we multiply 0.3536 times F2 and find that 0.3536 × 1126.135 is 398.20 lbs.

To figure F2z, we multiply 0.6124 times F2 and find that 0.6124 × 1126.135 is 689.65 lbs.

Whenever the value of a sub force grows in magnitude, the physical alignment of the cable causes the other sub member and resultant to change maintaining their same proportion.

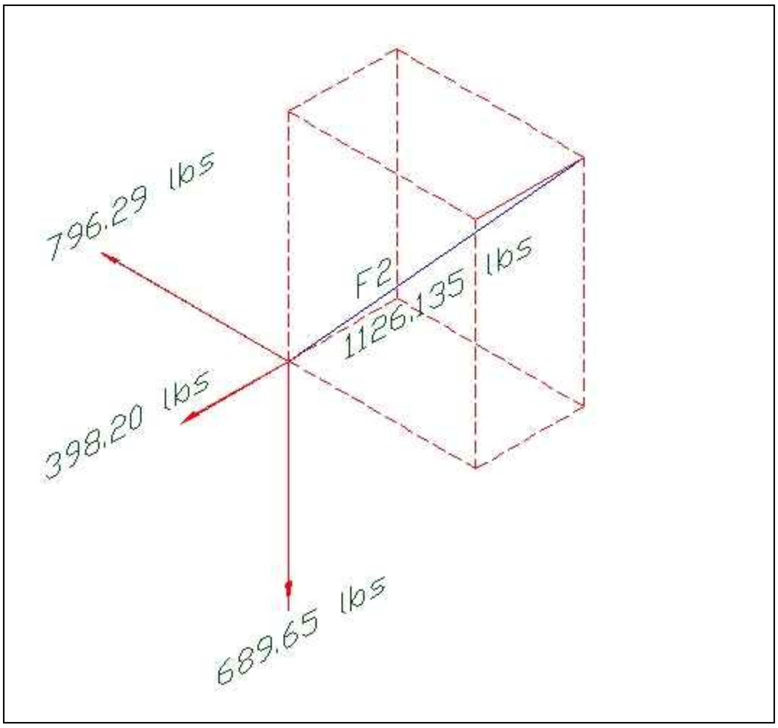


Figure 7.12 – Proportional Sub Forces of F2

Now, we will use the F3x quantity of 176.8 lbs and the F2x value of 398.20 lbs. to produce the answers for the tension in cable F1.

ΣFx = F1x – F2x – F3x = 0
F1x = F2x + F3x = 398.20 lbs + 176.8 lbs = 575 lbs

F1z equals F1x and to figure F1, we know that F1x equals 0.7071 F1, and then F1 equals F1x ÷ 0.7071. The resultant force of cable F2 is 575 lbs ÷ 0.7071 = 813.18 lbs.

Finally, we use the equation for the sum of the forces in the Z – direction to find the force of the pole on the foundation. We compute the force F3Z by the formula 0.6124 × F3 or 0.6124 × 500 lbs. which equals

ΣFz = F4z – F1z – F2z – F3z – Wt (pole) – Wt (guy lines) = 0
F4z = F1z + F2z + F3z + Wt (pole) + Wt (guy lines)
F4z = 575 lbs + 689.65 lbs + 306.2 lbs + 1961.54 lbs + 75 lbs = 3607.39 lbs

Using the CAD software gives the architect, designer or engineer a tool to quickly model an actual assembly, draw a force diagram, and to determine the delta X, Y and Z components of a resultant vector. In these last two problems, we also use the model to establish the volume of the assembly and the area in space that the structure blocked the wind. The next challenge is to practice what we have learned using a similar assembly that is only 30 feet tall. Complete the drawing of the model and the force diagram, and then write and solve the equations.

Practice Problem – Compute the Wind Load and Forces on a Pole

*** World Class CAD Challenge 10-15 * - Draw a 30 foot tall pole with a 12 inch diameter base. The hollow pole has a one inch thick wall at the bottom. The outside diameter extrudes at 0.5° and the inside diameter extrudes at 0.4°. Place an 80 mph wind load as shown in figure 7.13. The tension in cable F3 measures at 500 pounds.**

Continue this drill four times using some forces you have determined, each time completing the drawing under 120 minutes to maintain your World Class ranking.

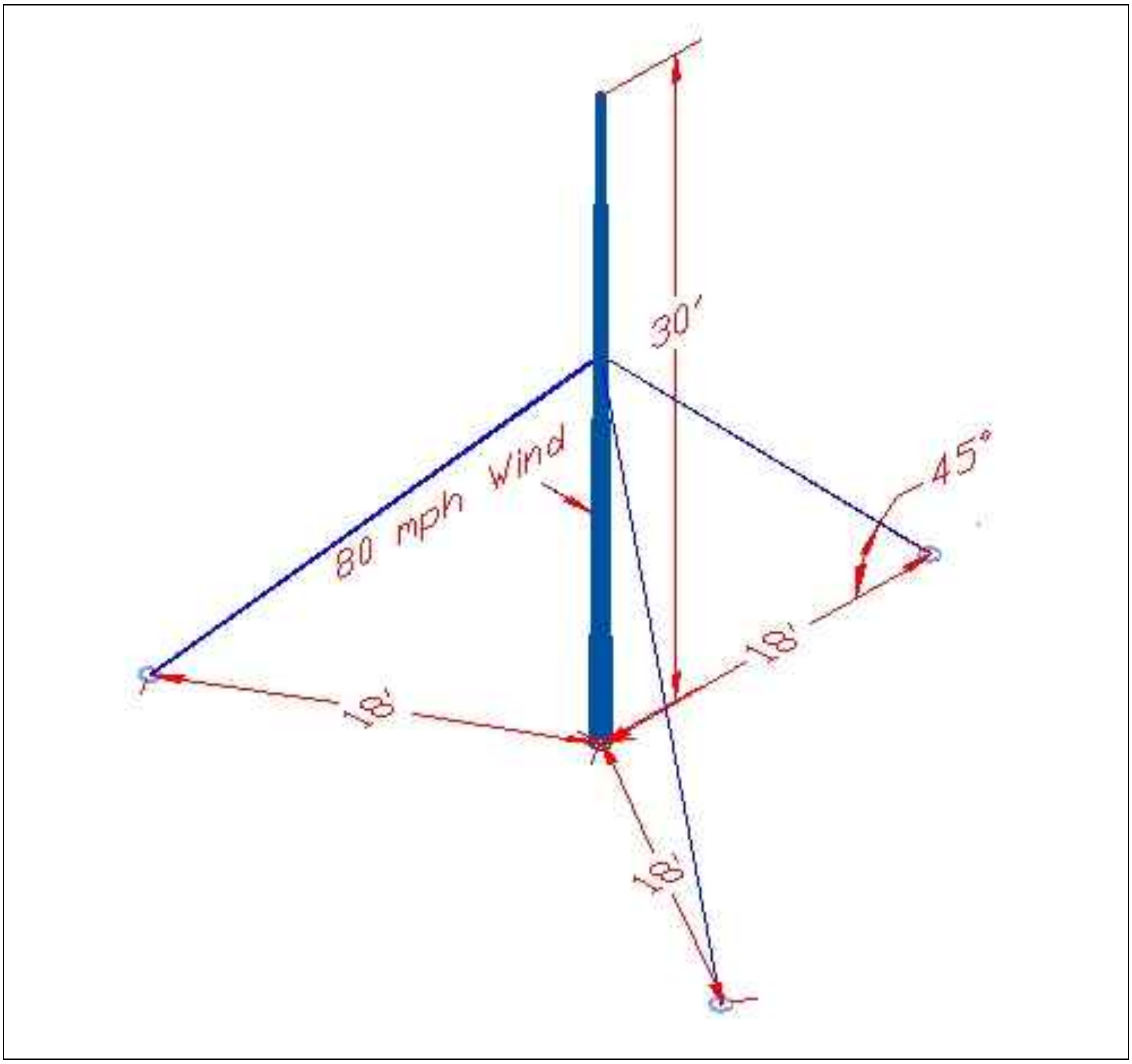


Figure 7.13 – Compute the Wind Load and Forces and a Pole