Chapter

6

Equilibrium in Two Dimensions

In this chapter, you will learn the following to World Class standards:

1. **The Ladder Against the Wall**
2. **The Street Light**
3. **The Floor Beam**

The Ladder Against the Wall

Beginning with this chapter, we will explore situations where the object is in equilibrium, where there is no movement of the part, and therefore we find the sum of the forces and the moments (torque) will equal zero. A structure at rest is a very common state for many products such as a microwaves, televisions or lampposts. We design these manufactured goods to stay still or at a constant velocity and as we say in engineering terms, remain static. In this section of the textbook, we will present conditions where there are forces such as cables, beams, wind and water that act on an entity, the result will be balance, and the assembly will continue to rest. After completing this unit, we will be able to determine by measurement all the external mechanical forces acting on the object.

In our first virtual experiment, we will place a 16-foot (192 inch) ladder against a wall. The ladder weighs 36 pounds (lbs) and we are trying to compute the magnitude and direction of the force on the ground and at the wall when a 350-pound force is 14 feet up the ladder. This would place a person with their tools at the second last rung and ready to step onto the roof of the building. This is a very practical problem and many designers and engineers will want to know the answers.

Replicating an experiment in lab. We can duplicate this entire problem in lab by scaling the project. We can make 1-foot equals 1-inch, so the ladder will be 16 inches long when we angle a small board representing the ladder at 65 degrees against a board. The weight of the small board measuring 0.125 x 1.0 x 16.0 will be negligible, so we place a 0.36 lb. weight, 8 inches up the ladder. We can place a 3.5 lb weight, 14 inches up the ladder.

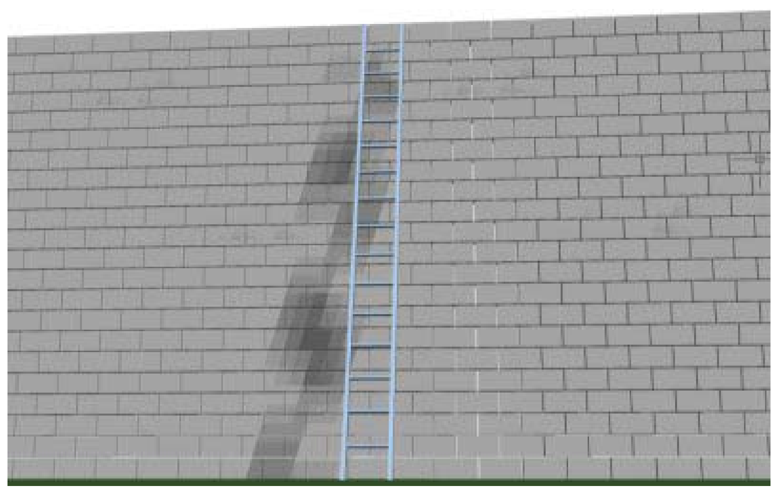

Figure 6.1 – Conducting the First Experiment

Our lab instructor will want to provide a measuring device to sense the weight pressing against the horizontal board that represents the wall. A small pressure scale will work fine, and we can compare the scaled answer, which is 1/10 of the real problem. Multiply the answer we get in the lab with the solution in this segment of the chapter to build our confidence to solve mechanical problems in the Computer Aided Design (CAD) program.

In solving for the unknown quantities in a Free Body Diagram, we will observe two fundamentals in the experiment. First, that the sum of the forces will equal zero. When we place the subcomponents of the forces in the graphic, the sum of the forces along each axis is zero. We write this mathematically using the summation symbol Σ, which is the Greek letter sigma. In Mechanics, we may see the following written in the calculation.

$$\sum F_X = 0$$
$$\sum F_Y = 0$$
$$\sum F_Z = 0$$

Which means the sum of the forces in the X direction is zero, the sum of the forces in the Y direction is zero and the sum of the forces in the Z direction is zero.

The other fundamental to maintain an object at rest or in uniform velocity besides the sum of the forces equals zero is that the sum of the moments (torques) will be equal to zero, too. This is written as:

$$\sum \tau_i = 0$$

Under these conditions, the body will not have an accelerated rotation, which would occur if the sum of the torques did not add up to zero. In our experiment, we can visually check when either fundamental rule is not occurring. In the research project shown in Figure 6.1, both fundamentals apply and the Free Body Diagram will solve for the forces on the wall and on the ground.

Now we will modify the experiment so we can examine the static problem with a Free Body Diagram. This type of illustration allows us to draw the force vectors that affect the object and use simple geometry to find the solution to the practical exercise. Then designers and engineers will apply the data in a couple of ways. If the other items in the environment control what shape the product will be, then the project designer or engineer will select a stronger material that has the appropriate characteristics to pass the qualification tests and survive the installation and continual usage throughout the years. Maybe, the engineer can change the form of the product using details like gussets and ribs to enhance the overall strength. Either way, the Free Body Diagram will tell us what forces are in the interacting members of the apparatus.

When computing moment arm, a measurement of force times distance, which is recorded in foot-pounds or inch pounds, we may project the force along the rigid body in the same line as the force vector. Thus, we will move the force to one of the axis so that the force is perpendicular to the Moment Arm we are trying to compute. At the 16-foot ladder, we are looking for a numeric value that is not 90º to the ladder, but to the wall, so we propel the line to the left towards the Y-axis. In Figure 6.2, the force is now at a right angle to the Y-axis.

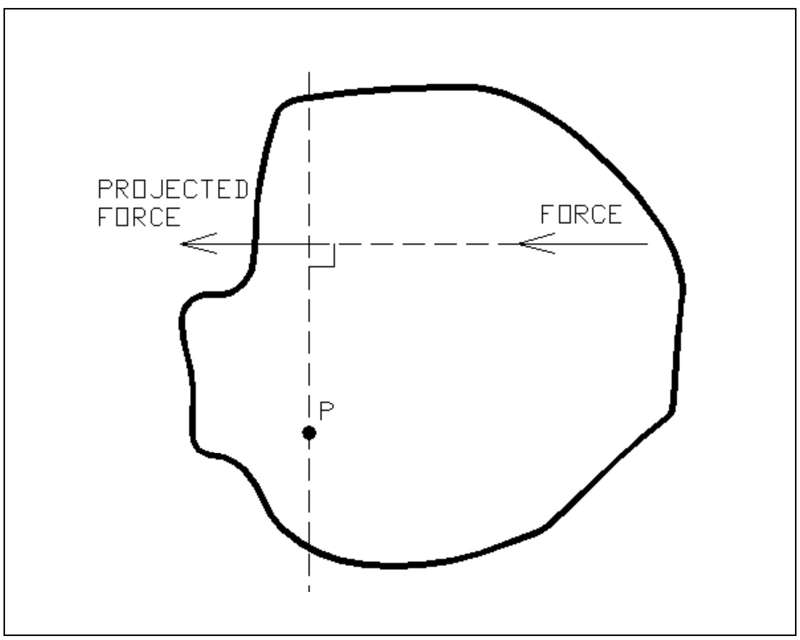

Figure 6.2 – Forces Perpendicular to the Moment Arm

When working with an equation that involves torque, we will consider the forces rotating counterclockwise as positive and those that are clockwise as negative.

In our Computer Aided Design (CAD) program, draw a two dimensional free body diagram showing the wall, the known forces we already mentioned, and label the two forces L1 and L2, which are the reactions to the load on the wall and on the ground. (See Figure 6.3) This problem has two unknowns, L1 and L2. First, we will discover the answer to L2, by solving for the sum of the forces and sum of the moments around L1. By using this technique, we only have to solve for one unknown at a time. After solving for L2, we will use the sum of the forces to determine the resultant at L1. Remember L1 will be equal in magnitude but opposite in direction to the resulting load at that point.

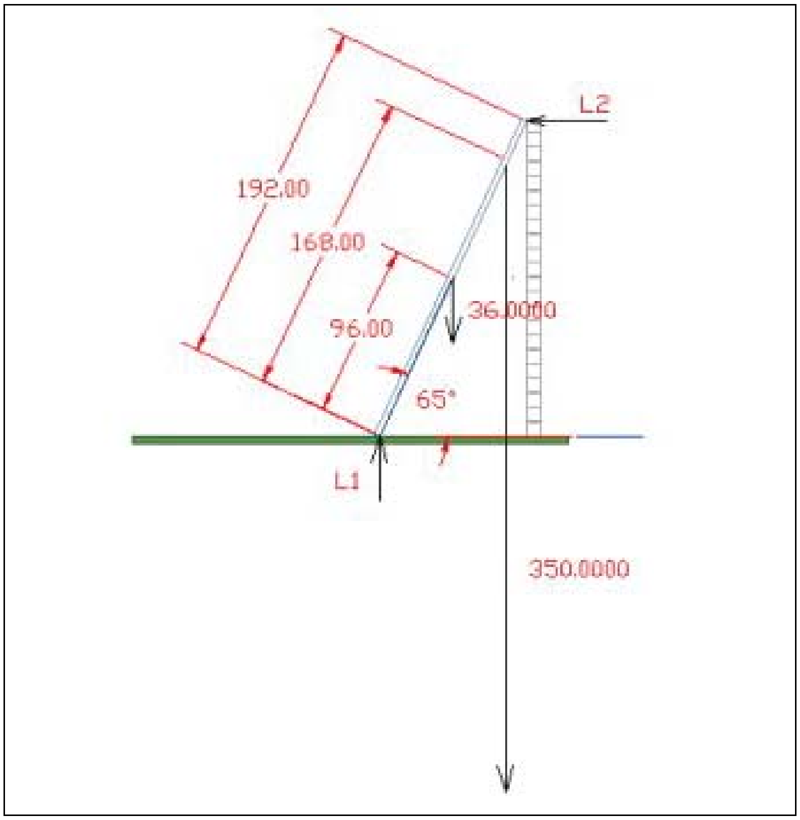

Figure 6.3 – CAD Drawing of the Ladder Problem

To find the unknown force L2, we are going to draw another diagram just to the right of the first (See Figure 6.4) and project the forces up along the X and Y-axis. The force L2 is 174.011 inches up the Y axis. The 350 pound force representing the person and tools and projected to the X axis is 69.7537 inches up the X axis. The weight of the ladder projected to the X axis is 40.5714 inches up the X axis. Now in order to find L2, we will multiply 350 times 69.7537, which equal 24413.795 inch pounds. Going in the same direction, we will multiply 36 times 40.5714, which equal 1460.5704 inch pounds. Add the two sums together and we get 25874.3654 inch pounds.

$$\sum \tau_i = 350 LBS \times 69.7537 IN. + 36 LBS \times 40.5714 IN. - L2 \times 174.011 IN = 0$$

$$25874.3654 IN \sim LBS. - L2 \times 174.011 IN = 0$$

$$L2 = 25874.3654 IN \sim LBS \div 174.011 IN = 148.6938 LBS$$

Now the sum of the Moment Arm to the right must be offset by the sum of the moment arm to the left. To find the value of L2, we will divide 25874.3654-inch pounds by 174.001 inches, which equals 148.6938 pounds. We know that the ladder has passed manufacturing tests to withstand 148.6938 pounds of force at the top of the ladder rails, but will the wall hold the load? If we did not know, we would run a test.

The next step is to use this number to determine the sum of the forces in the X and Y directions at point L1. The total of the forces in the X direction is 148.6938 pounds and the sum of the forces in the Y direction is 36.0000 and 350.000, which total 386.0000 pounds.

$$\sum F_Y = -36LBS - 350LBS + L1_Y = 0$$

$$L1_Y = 386.0000LBS$$

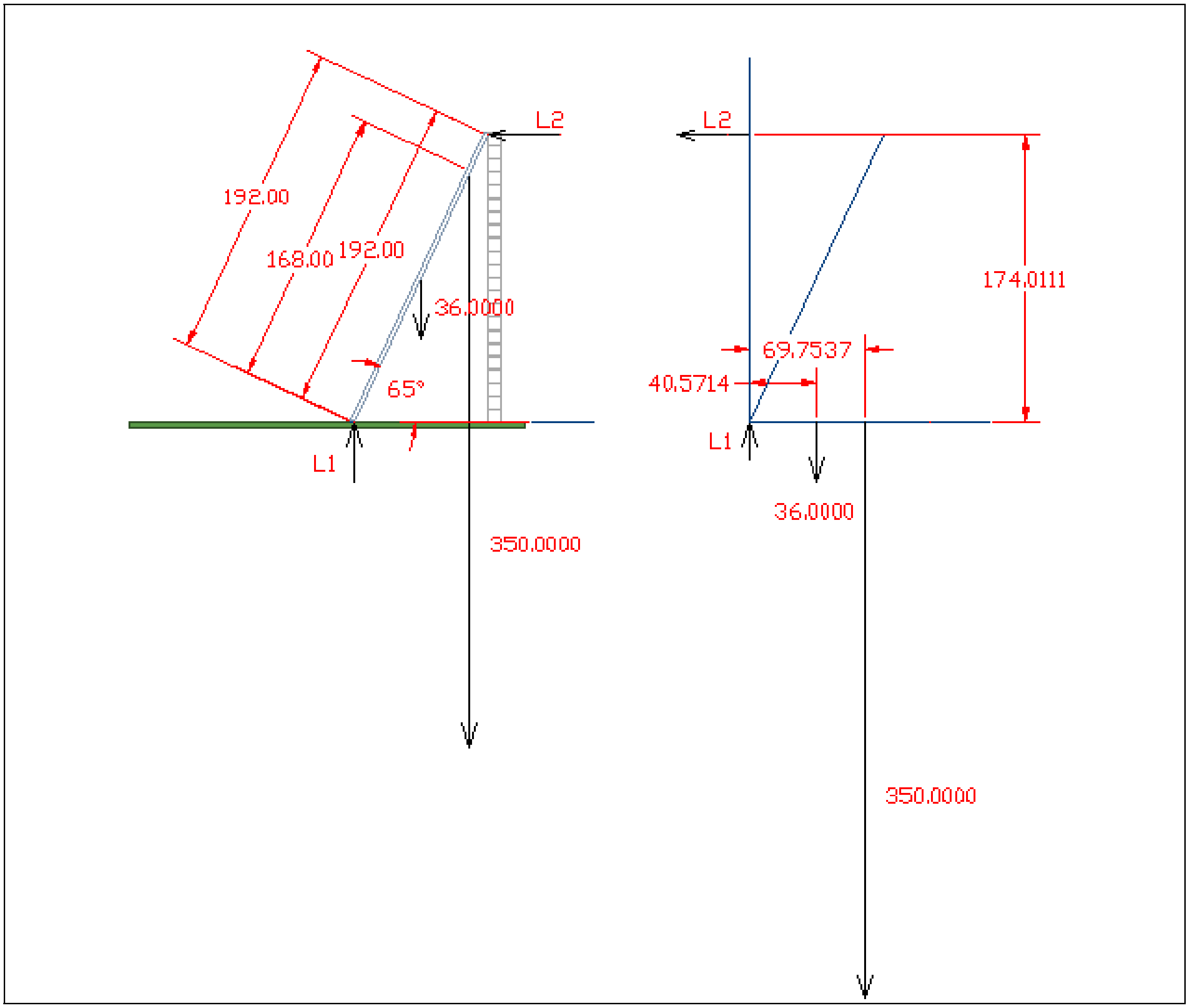

Figure 6.4 – The 2D Free Body Diagram

To find the resulting force that is resisting the ladder on the ground, we will also figure the force of L1 in the X direction. The sum of the forces in the X direction is:

$$\sum F_X = -148.6938LBS + L1_X = 0$$

$$L1_X = 148.6938LBS$$

We can use the CAD program to compute the forces acting on L1. In Figure 6.5, we draw the 386-pound force in the negative Y direction and 148.6938 in the negative X direction. Using vector addition, we draw a resulting vector and use the Align dimensioning tool on the Dimension toolbar to compute both the magnitude and orientation of the force that the ladder places on the ground. To discover the force L1, the figure will be equal in size and reverse in course by 180º. So the answer is 413.6494 pounds and 68.93º in above the X – axis.

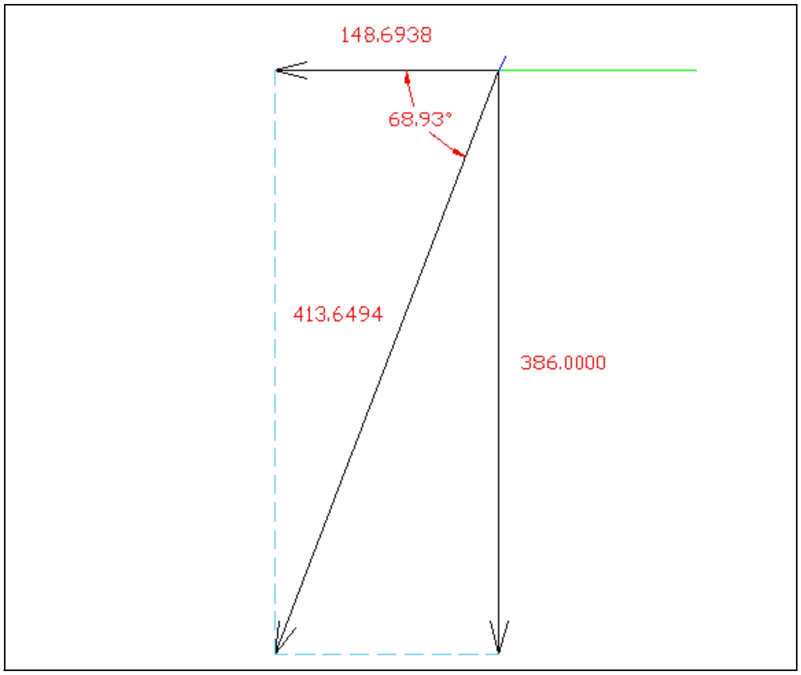

Figure 6.5 – The Resultant Force, L1

*** World Class CAD Challenge 10-12 * - Draw a 16 foot (192 inch) ladder leaning at 65º against a vertical wall Compute the forces L1 and L2 that resist the weight of the 36 pound ladder with a 350 pound load at 14 feet up the ladder. Save the drawing as Equilibrium Problem 5.dwg**

Continue this drill four times using some forces you have determined, each time completing the drawing under 5 minutes to maintain your World Class ranking.

The Street Light

The next exercise that we will analyze is a common application of a load being held by a cantilever arm outcropping from a vertical pole. A street light, a stop light, a crane or a plant hanger, are all structures that embrace a hanging load, where the beam is attached only on one end. In many cases, as in this exercise, a cable runs at an angle from the end of the cantilever beam to a position on the vertical pole, and thus the cable shares the weight.

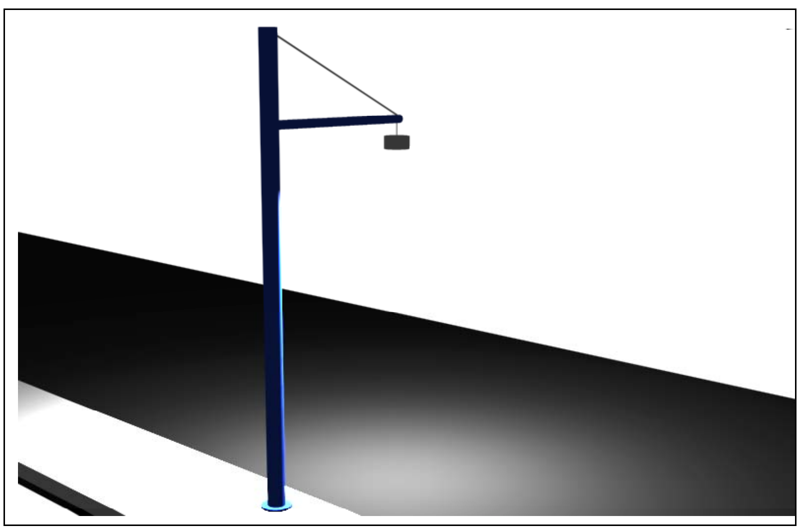

Figure 6.6 – The Street Light

In this problem, we will talk about three concepts in direction of forces on a material. The terms we want to know after this chapter are tension, compression and shear. Tension is a pulling action where the fibers of the material are stretching apart. Compression is a pushing action that has the strands of material coming together. Tension and compression are forces that are opposite in direction when we examine them in a detail. In this exercise, we do have a situation where the forces are acting in shear. When the cantilever arm holding the lamp intersects with the vertical pole, there is a shear force acting at 90 degrees to the cross section of the upright post. In later chapters, we will learn how to determine whether the slice of horizontal material in the pole will be able to withstand the shear load.

The arm coming out from the vertical pole in Figure 6.7 is in compression and the cable is in tension. The load going down the pole is equal to the load of the lamp. In our example, we will eliminate the weight of the cantilever arm itself and just concentrate of the heaviness of the lamp. Now there are many dimensions on the lamppost, but the only one we need to concentrate upon in this problem is the angle between the cable in tension and the cantilever arm in compression.

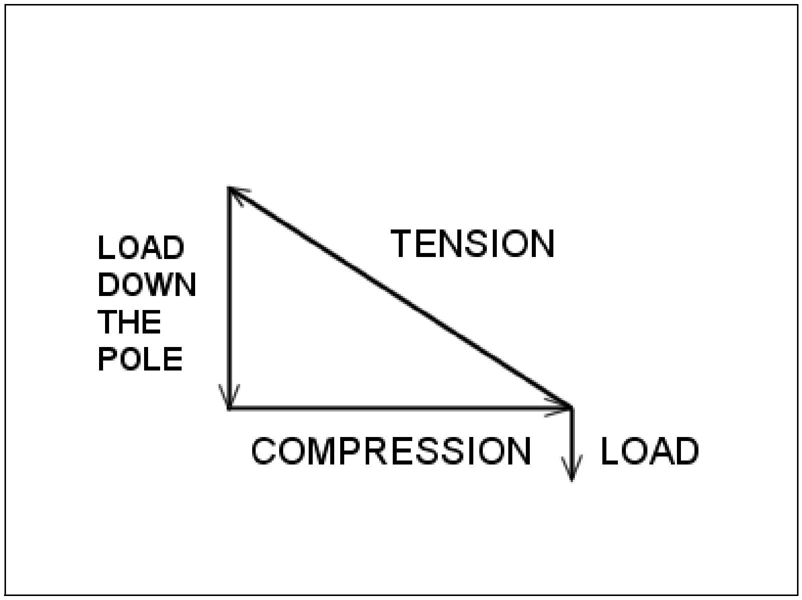

Figure 6.7 – Cantilever Load Diagram

The tension in the cable counteracts the vertical load of the lamp and the horizontal load of the cantilever arm. The angle between the cable in tension and the cantilever arm in compression is critical in this design. The larger the angle at the end of the triangle, the smaller the tension will be in the cable, and the compression in the cantilever arm, but the vertical pole will need to be taller. On the other extreme, the smaller the angle at the endpoint of the triangle, the larger the amount of tension in the cable and compression in the cantilever arm. What will happen in this type of design when the cable is removed? Where will the tension be located? Where will the compression force go?

In our Computer Aided Design (CAD) program, draw a 75-unit force vector downward. Draw a horizontal line to the left and a 26.6-degree line above the level line. Offset the horizontal line 75 units upward and extend the angled line so that they meet. Go ahead and erase the offset line. Drop a 75-unit line back down to the horizontal line and we will have a drawing similar to that as shown in Figure 6.8. Label each line and angle using our dimensioning tools. The tension in the cable for our problem is 167.7051 pounds and the compression in the beam is 150 pounds.

Replicating an experiment in lab. We can duplicate this entire problem in lab by scaling the project. We can make 0.1 pound equal 10 pounds, so the weight in the lab will be 0.75 pounds when we angle create the triangle. A heavy string or cord will easily hold the weight. The weight of the piece of wood acting as our cantilever arm will be negligible. Place the spring scale in line with the cable. As we measure the tension in the cable for the experiment, change the angle at the end of the triangle, so we can prove the relationship between the tension in the cable and the angle of intersection.

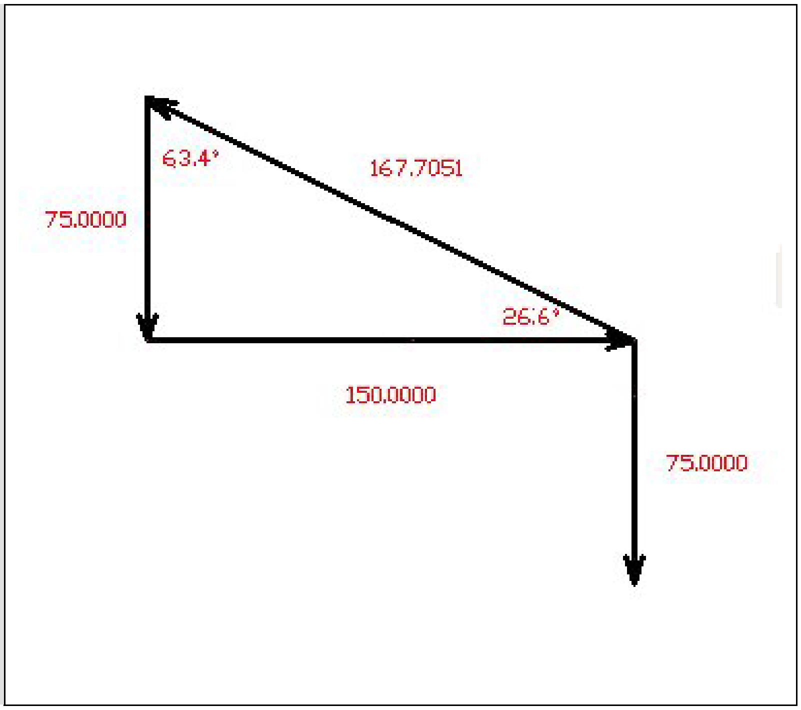

Figure 6.8 – Equilibrium Diagram

*** World Class CAD Challenge 10-13 * - Draw a lamppost that has a cantilever arm at least 14 feet above the ground. Attach a cable to the end of the arm and at 26.6º, extend the cable to the vertical pole. Compute the tension in the cable, the compression in the arm and the force transmitted down the lamppost. Save the drawing as Equilibrium Problem 6.dwg**

Continue this drill four times using some forces you have determined, each time completing the drawing under 5 minutes to maintain your World Class ranking.

The Floor Beam

In the next exercise, we will examine the forces on a floor beam like we would see in a house or on a deck. In Figure 6.9, we will set a 250-pound filing cabinet on the floor, with the center of the load being 60 inches or 5 feet from the left support. This floor has 2 x 10 floor joists that are sitting on 2 x 12 supporting beams spaced 144 inches or 12 feet apart. The floor joists are spaced 16 inches apart. We can observe in Figure 6.9 that beam problems are very common and practical engineering problem that we may face every day. Therefore, we must have the ability to perform these calculations without hesitation or error.

The techniques to reach a solution to the unknown forces are essentially the same as we did when solving the ladder exercise. We need to draw an equilibrium diagram, label the loads, the unknown resulting forces and the distances between the load and the supporting forces, R1 and R2. After making a diagram that is dimensionally correct, then we will solve for the resultant force R2 by examining the torque in the setup around the force R1. Since R1 is zero distance from the rotation point, the unknown value R1 is eliminated in this first inspection.

In Figure 6.10, we first draw the floor beam and filing cabinet in our CAD program. In this chapter, we will just analyze the resulting forces at the 2 x 12 supporting beams, but in later chapters, we will examine the cross section of the 12-foot long, 2 x 10 joists to determine whether they can carry such a load, or at what point they would fail. In the figure, we dimension the distances between entities using inches, but we could pick feet instead.

Figure 6.9 – Equilibrium Diagram

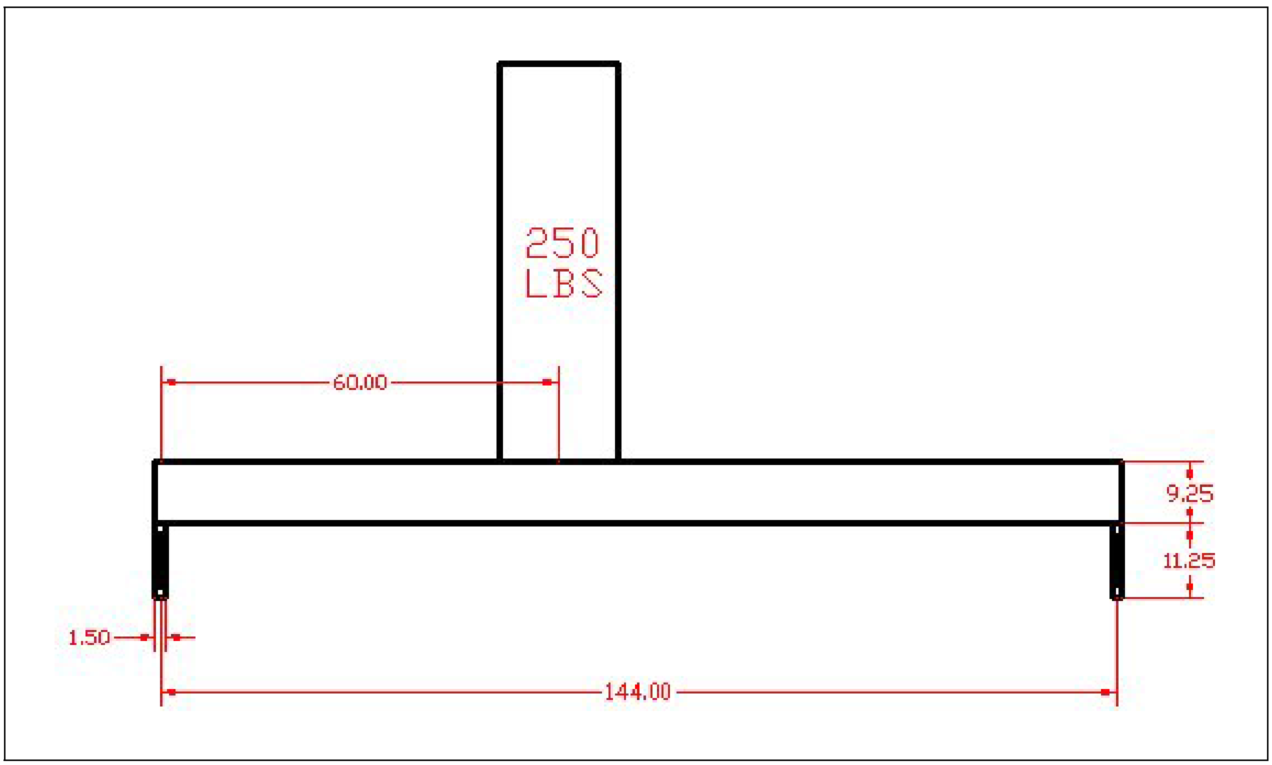

Figure 6.10 –Diagram of the Floor Beam

When we create the equation for the sum of the torque that is centered on the point at R1, which equals zero, we will multiple the 250 pounds times 60 inches and subtract the product of R2 times 144 inches. We write the problem as:

$$\sum \tau_i = 250LBS \times 60IN. - R2 \times 144IN. = 0$$

$$15000IN \sim LBS - R2 \times 144IN. = 0$$

$$R2 \times 144 = 15000$$

$$R2 = 15000IN \sim LBS \div 144IN. = 104.1667 \text{ LBS}$$

The sum of the forces in the floor beam problem is the total of R1 and R2 minus 250 pounds equal zero. We write the equation as:

$$\sum F_Y = R1 + R2 - 250 = 0$$

$$R1 + 104.1667LBS - 250LBS = 0$$

$$R1 = 250LBS - 104.1667LBS = 145.8333LBS$$

Replicating an experiment in lab. We can duplicate this entire problem in lab by scaling the project. We can make 1 pound equal 100 pounds, so the weight in the lab will be 2.5 pounds when we place the load on the beam. The weight of the piece of wood acting as our beam will be negligible. The horizontal beam can rest on two scales, so we can measure the forces at R1 and R2. You can move the load on the beam to develop an idea of what will happen when the weight is positioned differently along a beam.

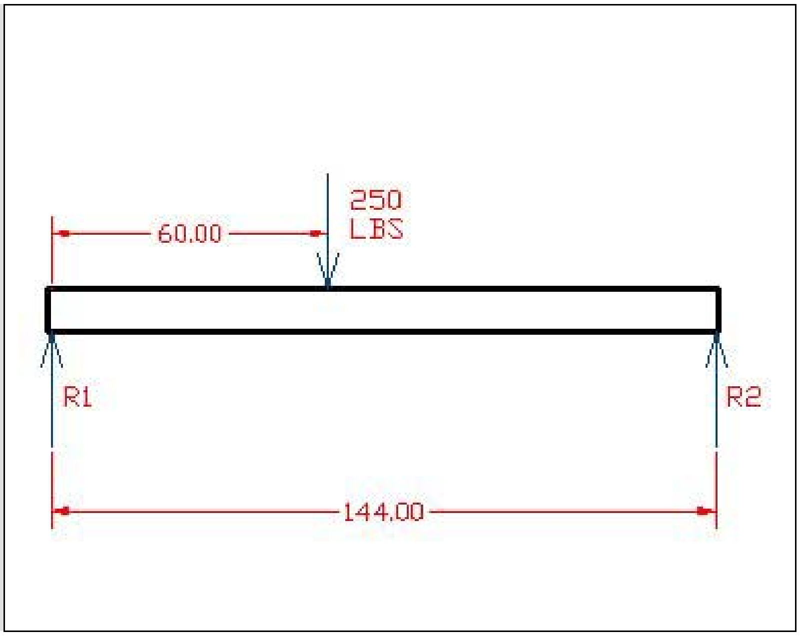

Figure 6.11 –Equilibrium Diagram

*** World Class CAD Challenge 10-14 * - Draw a 2 x 10 joist on 2 x 12 supports that are on 12 foot centers. Computer the resulting forces R1 and R2 for a 250-pound load. Save the drawing as Equilibrium Problem 7.dwg**

Continue this drill four times using some forces you have determined, each time completing the drawing under 5 minutes to maintain your World Class ranking.