C h a p t e r

5

Free Body Diagrams

In this chapter, you will learn the following to World Class standards:

1. **Measuring Force Results from Gravity Acting on a Mass**
2. **Solving a Mechanics Problem with a Free Body Diagram**
3. **Making Minor Changes in Mechanics to Reinforce Learning**
4. **Drawing a Free Body Diagram with Torque Equaling Zero**

Measuring Force Results from Gravity Acting on a Mass
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Beginning with this chapter, we will explore situations where the object is in equilibrium, which there is no movement of the part, and therefore we find the sum of the forces and the moments will equal zero. This is a very common state for many products such as a microwave, television or lamppost. We design these manufactured goods to stay still or at a constant velocity and we say in engineering terms, remain static, although the environment around them may act upon the assembly. In this section of the textbook, we will present conditions where there are forces such as cables, beams, wind and water that act on the entity and the result will be balance. After completing this unit, we will be able to determine by measurement all the mechanical forces acting on the object.

In our first exercise, we will build two columns and connect a beam running horizontally between the two pillars. At the center of the beam, we will hang a force meter with a can hanging beneath. We will fill the can with about 35 cubic inches of sand, and stop when the gauge reads 2.0 pounds.

What causes the gauge to read the mass of the sand? One mass, which in your vicinity is the Earth, is pulling the sand downward toward its center. The sand, which also has mass has an attraction on objects in the area, but not as large as the Earth's gravity well. We can measure the effect will a spring scale that has been calibrated to read in pounds. The weight or force measurement is significant in the study of Statics, but later when the object is moving, we will concerned with the mass of the body.
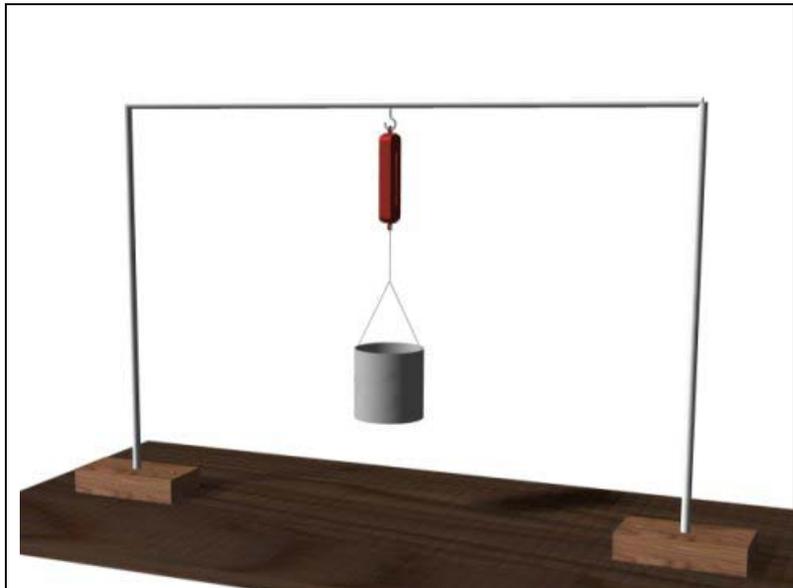
Figure 5.1 – Conducting the First Experiment

The formula to compute the mass of an object is:

$$w=mg$$

where

w – weight
m – mass
g – gravity

The gravity constant we will use in our exercises is 32.2 feet per second per second. Notice the figure for gravity is the same form as acceleration.

We are using a principle, which demonstrates that for any action (force) being applied, another action (force) equal and opposite of the first will occur. Using Isaac Newton's Third Law of Motion, in Figure 5.2, we see that the force exerted by the sand falling towards the Earth is opposed by the cable holding the can and attached to the beam. The gauge shows the measurement of 2.0 pounds.

In our Computer Aided Design (CAD) program, draw an exact replica of the object we are using in the experiment. Construct a force vector that is two units long and pointing downward in the negative Z direction. We need to dimension or label the vector. Create another force vector at the top of the can, pointing in the positive Z direction that is two units long and label the entity as shown in Figure 5.2.
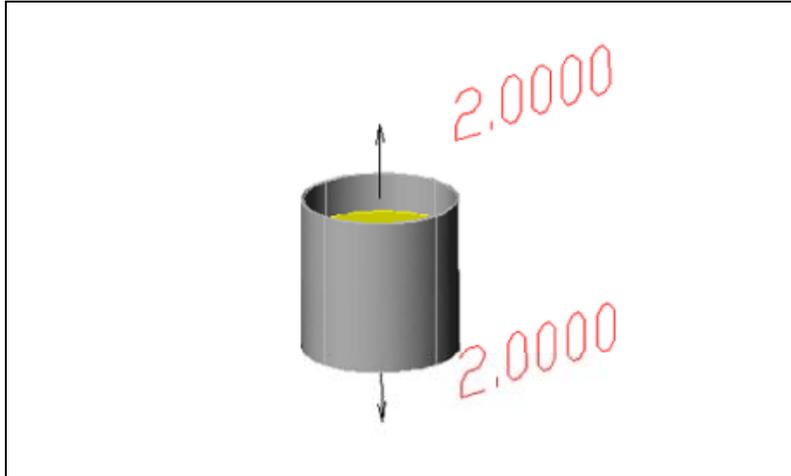
Figure 5.2 – A Can of Sand in Equilibrium

* World Class CAD Challenge 10-8 * - Draw a solid can containing a solid cylinder representing the amount of sand within. Illustrate a 2.0-pound force vector at 90 degrees above the XY plane and a 2.0-pound force vector at 90 degrees below the XY plane. Save the drawing as Equilibrium Problem 1.dwg

Continue this drill four times using some forces you have determined, each time completing the drawing under 5 minutes to maintain your World Class ranking.

Solving a Mechanics Problem with a Free Body Diagram

Now we will modify the experiment so we can examine the static problem with a Free Body Diagram. This type of illustration allows us to draw the force vectors that affect the object and use simple geometry to find the solution to the practical exercise. Then designers and engineers will apply the data in a couple of ways. If the other items in the assembly very closely determine the shape of the product, then the project manager will select a material that has the appropriate strength to pass the qualification tests and survive installation and usage throughout the years. Maybe, the engineer can change the form of the entity using detail to enhance the overall strength. Either way, the Free Body Diagram will tell us what force is in the interacting members of the apparatus.

In solving for the unknown quantities in a Free Body Diagram, we will observe two fundamentals in the experiment. First, that the sum of the forces will equal zero. When we place the subcomponents of the forces in the graphic, the sum of the forces along each axis is zero. We write this mathematically using the summation symbol Σ, which is the Greek letter sigma.

In Mechanics, we may see the following written in the calculation.

$$\sum F_X = 0$$
$$\sum F_Y = 0$$
$$\sum F_Z = 0$$

Which means the sum of the forces in the X direction is zero, the sum of the forces in the Y direction is zero and the sum of the forces in the Z direction is zero.

The other fundamental to maintain an object at rest or in uniform velocity is that the sum of the torques will be equal to zero.

$$\sum \tau_i = 0$$

Under these conditions, the body will not have an accelerated rotation, which would occur if the sum of the torques did not add up to zero. In our experiment, we can visually check when either fundamental rule is not occurring. In the research project shown in Figure 5.3, both fundamentals apply and the Free Body Diagram will solve for the tension in the cable.

In our second exercise, we will use the beam running horizontally between the two pillars build two columns setup and connect a hook on the left side holding the force meter and running the cable to a hook on the right side of the beam. At the center of the cable, we will hang our can, which we previously weighed at 2.0 pounds. Now using a protractor, arrange the cable so that we read 30 degrees between the cable and the horizontal beam. Check both sides of the cable for accuracy.
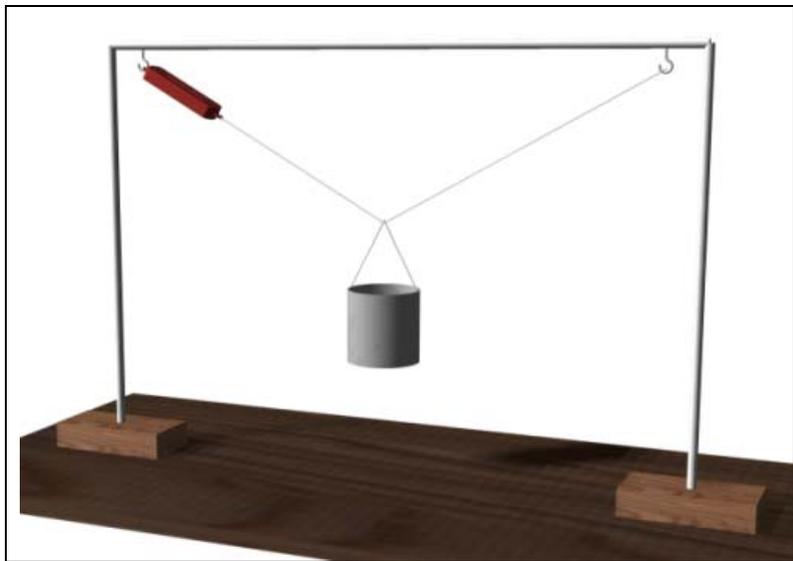
Figure 5.3 – The Second Equilibrium Experiment

Next, in our CAD program, draw an exact replica of the object we are using in the experiment. Construct a force vector that is two units long and pointing downward in the negative Z direction. We need to dimension or label the vector. Create another force vector at the top of the can, pointing up 30 degrees above the XY plane in the positive Z direction. We do not know what the measurement of the force is now so we will make the vector whatever distance we choose. Create a third force vector at the top of the can, pointing up 30 degrees above the XY plane in the positive Z direction and 180 degrees opposite the second vector. Again, we do not know what the size of the force is now so we will make the vector whatever distance we decide. Our diagram should appear in the CAD software application as shown in Figure 5.4

To one side in the CAD program, construct the Free Body Diagram as shown in Figure 5.5. We can see that both cables will share the load equally since the problem is symmetrical, therefore we will draw a small triangle just below the Free Body Diagram. We will examine just one side of the problem, and draw a 1.0-pound line vertically.
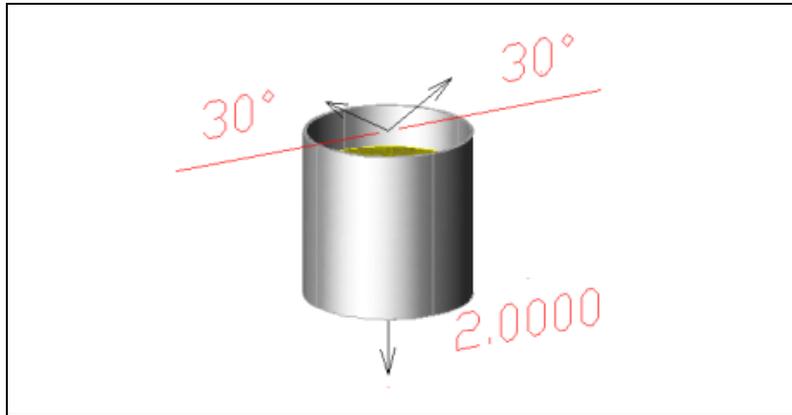
Figure 5.4 – Drawing the Apparatus in CAD

Then, we draw line 30 degrees above the XY plane at an undetermined length. Draw a third line from the top of the vertical 1.0-pound line to the right at any distance. Use a CAD tool to trim the lines or join them together to form a perfect 30-60-90 triangle. The true length of the hypotenuse of the triangle in this exercise is the tension in the cable.
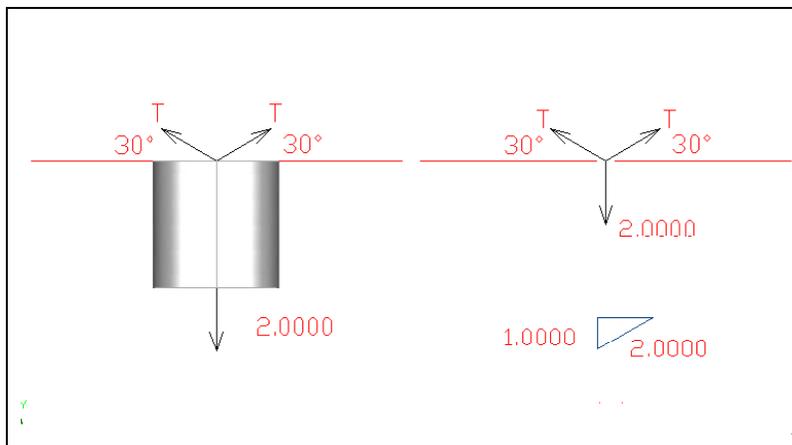
Figure 5.5 – Drawing the Free Body Diagram

Using the Align Dimension tool, we find the true length is 2.0. Now check your force meter in experiment two. They are very close, so adjust the two vectors in your Free Body Diagram to read 2.0 pounds. Will the cable tension always equal the weight on the cable? We need to continue to discover answers to these mechanical problems in experiment three.

* World Class CAD Challenge 10-9 * - Draw a solid can containing a solid cylinder representing the amount of sand within. Illustrate a 2.0-pound force vector at 90 degrees below the XY plane and two counteracting force vectors at 30 degrees above the XY plane. Determine the size of the two force vectors at 30 degrees above the XY plane. Save the drawing as Equilibrium Problem 2.dwg

Continue this drill four times using some forces you have determined, each time completing the drawing under 5 minutes to maintain your World Class ranking.

Making Minor Changes in Mechanics to Reinforce Learning
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In the next exercise, we need to make minor changes to examine what occurs in the world

where we live and whether we have the CAD skills to simulate the problem. In the previous experiment, we were able to draw a Free Body Diagram and use the triangle, a figure representing balance, to find the unknown tension in the cable. Can we assume that the tension in two cables holding a weight symmetrically will always equal the force pulling downward? That will make our lives as designers and engineers very easy.

In our third exercise, we will again use the beam running horizontally between the two pillars build two columns setup and connect a hook on the left side holding the force meter and running the cable to a hook on the right side of the beam. At the center of the cable, we will hang our can, which we previously weighed at 2.0 pounds. Now using a protractor, arrange the cable so that we read 20 degrees between the cable and the horizontal beam. Check both sides of the cable for accuracy.
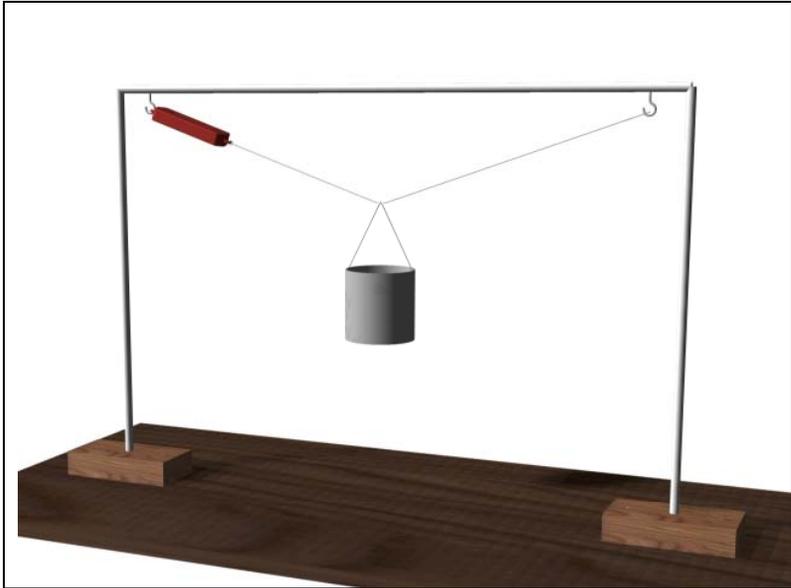
Figure 5.6 – The Third Equilibrium Experiment

Save the previous CAD Free Body Diagram and Save As again to rename the drawing as Experiment 3. We need to change both of the force vectors at the top of the can that is pointing up 30 degrees above the XY plane in the positive Z direction, to 20 degrees. We do not know what the measurement of the force is now so we will make the vector whatever distance we choose. Our diagram should appear in the CAD software application as shown in Figure 5.7.

To one side in the CAD program, construct the Free Body Diagram as shown in Figure 5.7. We can see that both cables will share the load equally since the problem is symmetrical, therefore we will draw a small triangle just below the Free Body Diagram. We will examine just one side of the problem, and draw a 1.0-pound line vertically.

Then, we draw line 20 degrees above the XY plane at an undetermined length. Draw a third line from the top of the vertical 1.0-pound line to the right at any distance. Use a CAD tool to trim the lines or join them together to form a perfect 20-70-90 triangle. The true length of the hypotenuse of the triangle in this exercise is the tension in the cable.
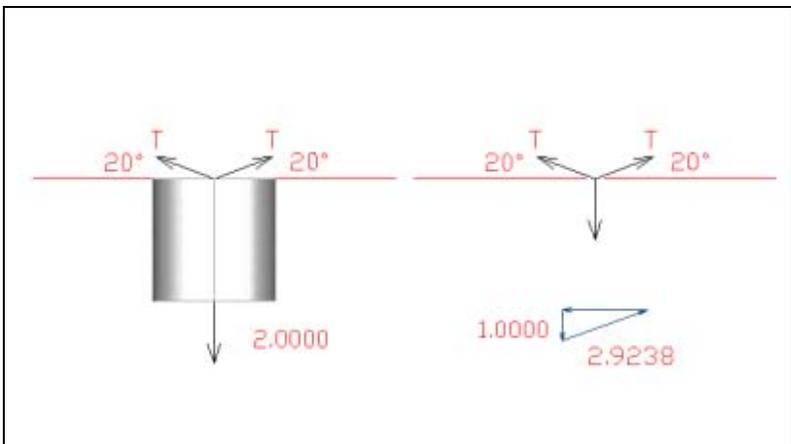
Figure 5.7 – Drawing the Free Body Diagram

Using the Align Dimension tool, we find the true length is 2.9238. Now check your force meter in experiment three. They are very close, so adjust the two vectors in your Free Body Diagram to read 2.9238 pounds. We can postulate that the two cables holding a load will have 1.0 pound tension each when hanging vertically or at 90 degrees, to 2.0 pounds tension each at 30 degrees, to almost 3.0 pounds tension each at 20 degrees. What would we expect the tension in the cables when they are 5 or 2.5 degrees from the horizontal beam?

* World Class CAD Challenge 10-10 * - Draw a solid can containing a solid cylinder representing the amount of sand within. Illustrate a 2.0-pound force vector at 90 degrees below the XY plane and two counteracting force vectors at 20 degrees above the XY plane. Determine the size of the two force vectors at 20 degrees above the XY plane. Save the drawing as Equilibrium Problem 3.dwg

Continue this drill four times using some forces you have determined, each time completing the drawing under 5 minutes to maintain your World Class ranking.

Drawing a Free Body Diagram with Torque Equaling Zero

In the exercise, we will examine the roll of torque in an assemblage of objects. One of the easiest ways to view an example of the concept is through a seesaw. One weight is affecting the system in a counterclockwise direction around the rotation point and the other weight is affecting the system in a clockwise direction around the rotation point. In our Free Body Diagram, we will want the system to balance.

In our fourth experiment, we will use a wooden board measuring 0.5 x 3.5 x 24.0 sitting on a fulcrum. At one end of the board, place the can, which we previously weighed at 2.0 pounds. At the other end of the board, place another can, which is empty. Measure 8.0 inches from the fulcrum to the center of the 2.0 pound can and your test setup will appear as shown in Figure 5.8.
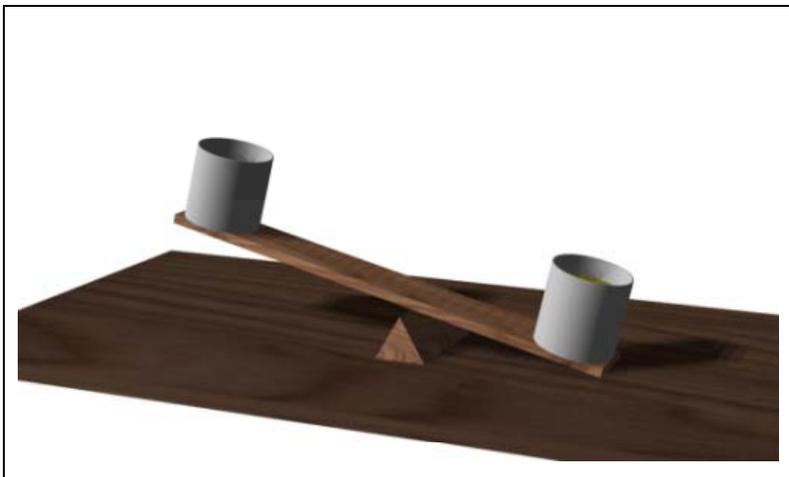
Figure 5.8 – Two Force Vectors Acting on an Object

In our CAD application program, we will start to draw a Free Body Diagram by opening a new drawing and saving the file as Experiment 4. Draw a replica of the components used in the test, copying and pasting the 2.0 pound can of sand from the previous drawing. Draw the wedge we are using as a pivot and the 0.5 x 3.5 x 24.0 board.

Above the virtual experiment in the CAD program, construct the Free Body Diagram as shown in Figure 5.9. We will draw a 2.0 unit long vector, pointing in the negative Z direction, and is

8.0 inches from the fulcrum. We will draw a vector of any length for the unknown weight, pointing in the negative Z direction, and is 12.0 inches from the opposite side of the fulcrum. Dimension the two distances, 8.00 and 12.00. Label the two force vectors.

To find the weight that will balance the system, we will first measure the torque or Moment being applied by the 2.0-pound can by multiplying the force times the distance from the axis of rotation. We measure a Moment with both of these components, a force vector and the moment arm, which is the perpendicular distance from the point of rotation to the force vector. The Moment on the right side of the system shown with a clockwise angular arrow is 2.0 times 8.0 or 16 inch-pounds.
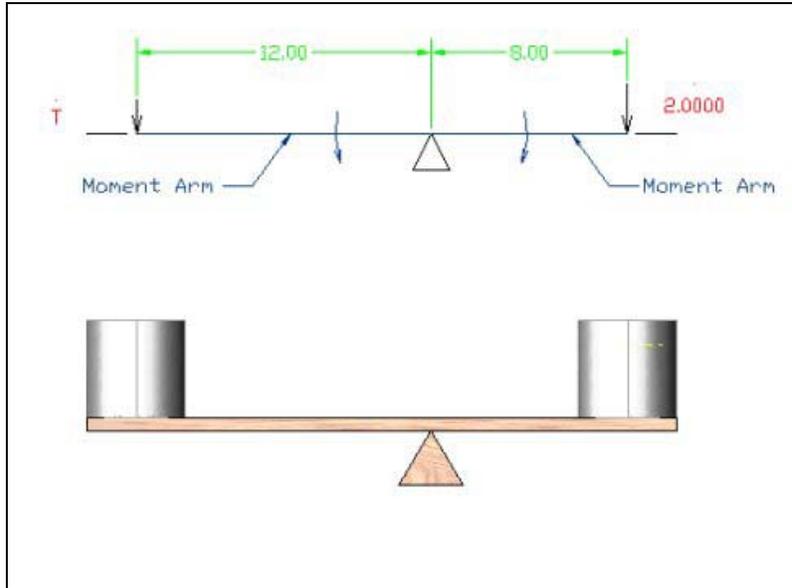
Figure 5.9 – Two Force Vectors Acting on an Object

To discover the unknown weight to balance the system, we will compute the Moment equation again and solve for the unknown quantity.

$$M = Fs$$

where

M – moment
F – force
s – distance

Now on the left hand side of the system, we need a Moment equal and opposite to the 16.0 inch-pounds clockwise torque. Therefore, we write the formula below.

16.0 inch pounds = F × 12.00 in

And we solve for the force, F.

F = 16.0 inch pounds ÷ 12.00 in = 1.33 pounds

Change the force vector length in the Free Body Diagram to 1.33 units long and label the vector with a dimension showing the measurement.

Fill the empty can with sand until the beam system is in balanced state. Remove the can and measure the weight with the force meter. Did we find the answer to be very close or entirely correct?

* World Class CAD Challenge 10-11 * - Draw a solid board that is 0.5 inch by 3.5 inch by 24.0 inch. Illustrate a 2.0-pound force vector at 90 degrees below the XY plane that is 8.0 inches from the rotation point. Illustrate a 2.0-pound force vector at 90 degrees below the XY plane that is 12.0 inches from the rotation point on the opposite side of the first force vector. Determine the size of the second force vector by using the formula M = Fs. Save the drawing as Equilibrium Problem 4.dwg

Continue this drill four times using some forces you have determined, each time completing the drawing under 5 minutes to maintain your World Class ranking.

By completing these experiments, drawing the Free Body Diagrams and finding the answers to the problems, we now have new capabilities in the world of design and engineering. We are using the CAD program that can clearly display the problem we want to analyze. We use the geometric functions and dimensioning tools to complete the mathematics. Did we see that the Moment is similar to the area of a rectangle formula? How can we use the area tool in your CAD program to substitute for the math we did with **M = Fs** and solve for the unknown force? Yes, we can do this technique. There will be other times in our engineering experience, where finding the area under a curve will give us important information, so continue with our experimenting.

After completing this chapter, do the addition equilibrium problems in Appendix B and then continue to Equilibrium in Two Dimensions in chapter 6.