Chapter 11

Moment of Inertia

In this chapter, you will learn the following to World Class standards:

- **What Makes the Beam Stronger**
- **Defining the Moment of Inertia**
- **Computing the Moment of Inertia with CAD**
- **Computing the Moment of Inertia for Hollow Shapes**
- **Computing the Moment of Inertia for Irregular Shapes**

What Makes the Beam Stronger

What makes a beam stronger? Does the orientation of the shape make a difference in resisting bending or breaking? We believe that many individuals already can make judgments based upon their life experiences as to how strong a beam is or what direction the shape should face when loading a beam.

Many of us have seen the floor joist in a house under construction or while in an unfurnished basement. We know that the 2 × 10 floor joist is stronger when the board stands on the 1.5 wide end. We select the wrench with a wider handle when the bolt head is bigger.

Beam	**Width of the Shape**	**Height of the Shape**
2 × 10 Floor Joist	*1.5*	9.25

Figure 11.1 – Researching the Strength of a Beam

We went to a worksite and measured a floor joist. Then we placed the data into the table shown in Figure 11.1. Your assignment is to research twelve more beams, get their measurements and place the data into the table. Share your information you have found with the other individuals conducting this exercise and with your instructor. Look for similarities and differences and write down any questions that you have regarding the construction of beams. Ask the instructor those questions when that subject is covered during this chapter.

Defining the Moment of Inertia

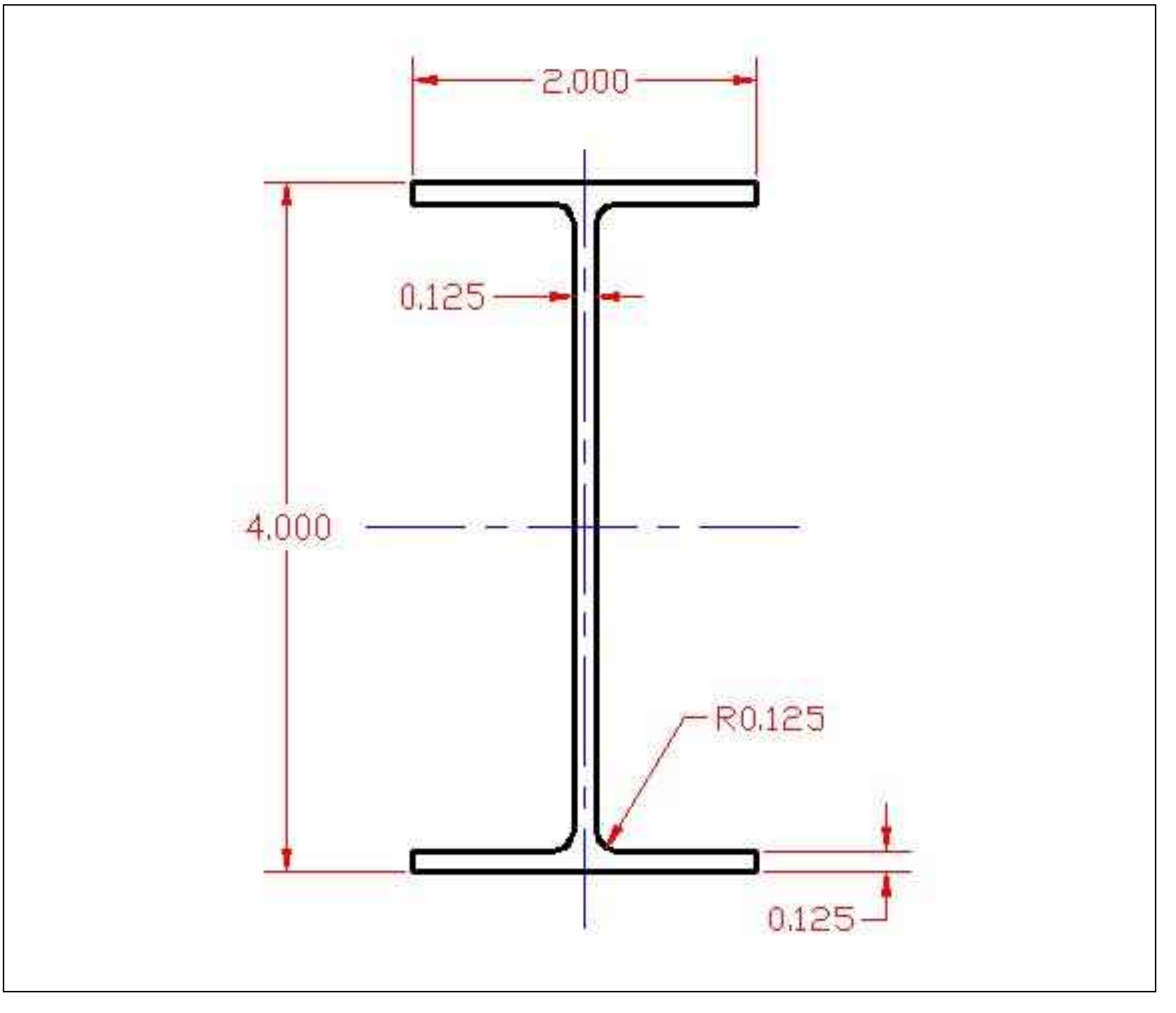

Figure 11.2 – Aluminum I-beam

The Moment of Inertia of an object is the ability of the shape to resist rotation around an axis. We define the Moment of Inertia by taking the sum of the cross sectional area of the beam and multiplying that number times the square of the radius to the center of the area.

The formula to find the Moment of Inertia for a rectangle is

$$I = \sum m_i r_i^{\,2}$$

To simplify the calculation of our practical problem, a symmetrical beam shown in Figure 11.2, we will take use multiple Moment of Inertias on the beam to come to an accurate estimate of the Moment of Inertia for the side above the center of gravity. We will multiply the sum of the Moment of Inertias by two to get our final answer.

We show the flange as the first area that we will define in Figure 11.3, where the measurement of the region is 2.000 by 0.125 and the distance from the center of the mass to the center of the beam is 1.9375.

$$I = Ar^2 = l \times w \times r^2$$

$$I = 2.000 \times 0.125 \times (1.9375)^2$$
$$= 0.938\text{in}^4$$

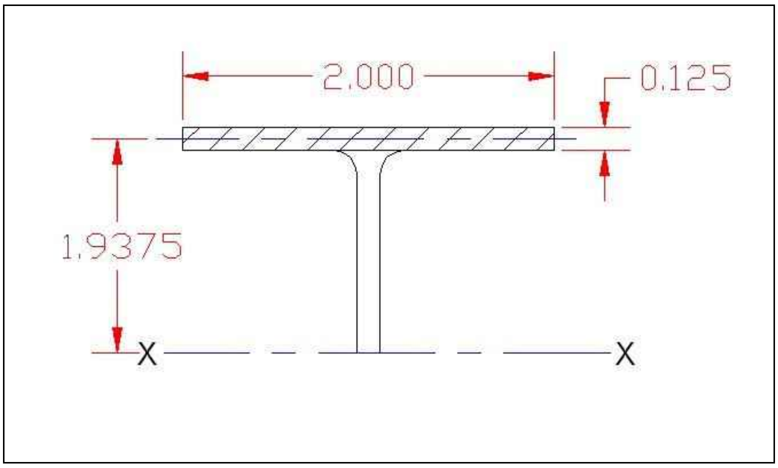

Figure 11.3 – Moment of Inertia of Area 1

For the web section of the I-beam, we divide the area into three parts to get better accuracy. The second area we will define in Figure 11.4, where the area of the region is 0.125 by 0.625 and the distance from the center of the mass to the center of the beam is 1.563.

$$I = Ar^2 = l \times w \times r^2$$

$$I = 0.125 \times 0.625 \times (1.563)^2$$
$$= 0.191\text{in}^4$$

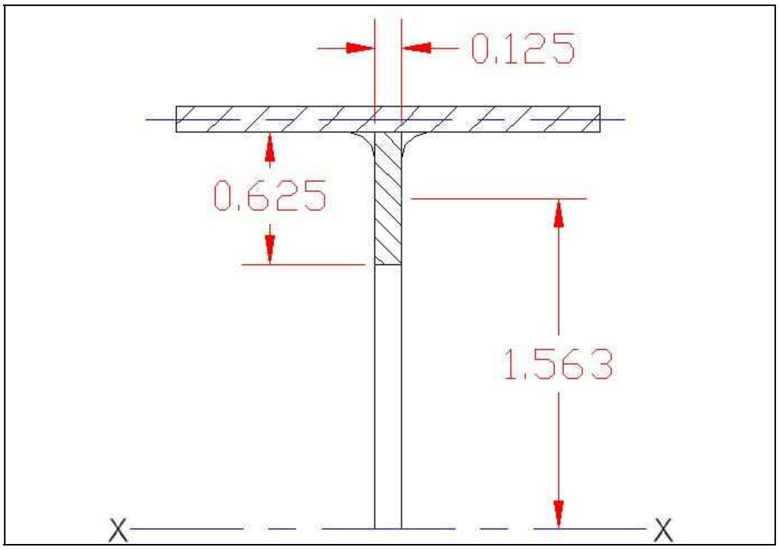

Figure 11.4 – Moment of Inertia of Area 2

The third area we will define is shown in Figure 11.5, where the area of the region is 0.125 by 0.625 and the distance from the center of the mass to the center of the beam is 0.938.

$$I = Ar^2 = l \times w \times r^2$$

$$I = 0.125 \times 0.625 \times (0.938)^2$$
$$= 0.069\text{in}^4$$

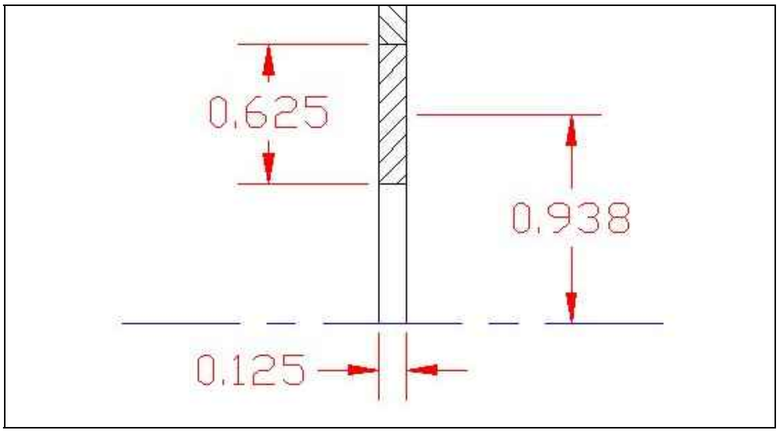

Figure 11.5 – Moment of Inertia of Area 3

The fourth area we will define is shown in Figure 11.6, where the measurement of the region is 0.125 by 0.625 and the distance from the center of the mass to the center of the beam is 0.313.

$$I = Ar^2 = l \times w \times r^2$$

$$I = 0.125 \times 0.625 \times (0.313)^2$$
$$= 0.008\text{in}^4$$

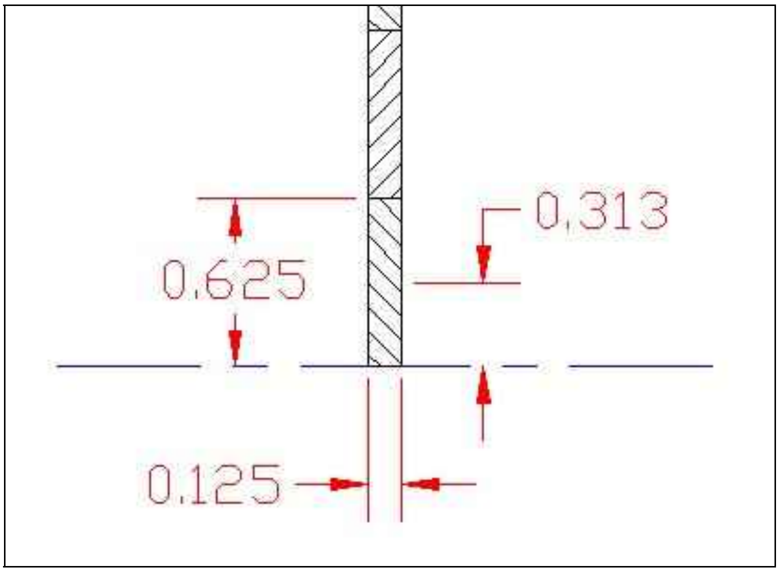

Figure 11.6 – Moment of Inertia of Area 4

For the last area, area 5, the filleted inner corner, we multiply the height and width of the square and subtract the area of one fourth of the circle. Use the following formula to find the area of the fillet.

$$A = 2 \times \left(h \times w - \frac{\pi r^2}{4} \right)$$

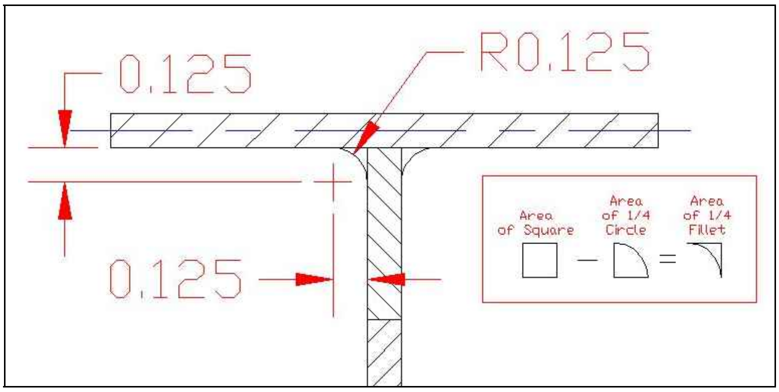

Figure 11.7 – Moment of Inertia of Area 5

Then, we insert real numbers for the variables below.

$$A = 2 \times \left(0.125 \times 0.125 - \frac{\pi (0.125)^2}{4} \right) = 2 \times (0.0156 - 0.0123) = 0.0066in^2$$

To find the Moment of Inertia of area 5, we take the area of the fillet and multiply that number by the square of the distance to the feature, which is 1.750 inches.

$$I = 0.0066 \times (1.750)^2 = 0.020\text{in}^4$$

Now, we total all five areas together to get a very close estimation of the Moment of Inertia of the I-beam.

$$I_{total} = 2 \times \left(I_{area1} + I_{area2} + I_{area3} + I_{area4} + I_{area5} \right)$$

And insert the real numbers of the five different Moment of Inertias

$$I_{total} = 2 \times \left(0.938\text{in}^4 + 0.191\text{in}^4 + 0.069\text{in}^4 + 0.008\text{in}^4 + 0.020\text{in}^4\right)$$

$$I_{total} = 2 \times \left(1.226\text{in}^4\right) = 2.452\text{in}^4$$

That is the manual technique we would use in Engineering school to calculate the Moment of Inertia using a sketchpad and a calculator. However, we should be using more modern tools like a Computer Aided Design program, which will give us the Moment of Inertia in seconds.

Computing the Moment of Inertia with CAD

Draw the I-beam from Figure 11.2 in your CAD program. We are using AutoCAD to draw and then calculate the Moment of Inertia of the shape. In our other textbooks and on the World Class CAD website, we have used the Mass Properties tool to measure volume and the Moment of Inertia of 3D solids. However, in this project, the drawing is two-dimensional and we will have to use the Region tool on the Draw toolbar to help us make the drawing ready for analysis.

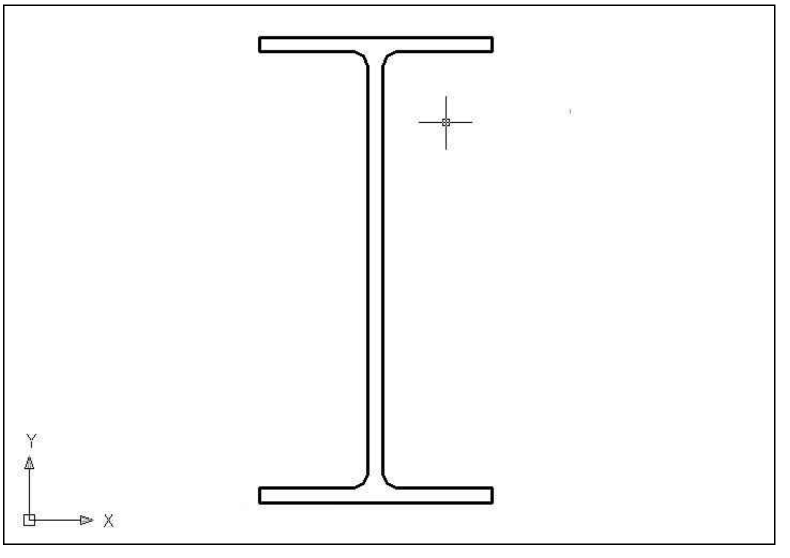

Figure 11.8 – I-beam in CAD

Select the Region tool on the Draw toolbar and choose all of the lines and arcs in the I-beam. Press Enter to convert the set of lines and arcs to a region.

A CAD region is a closed area such as the I-beam drawing as we see in Figure 11.9. In other projects in this chapter, we will remove interior areas from the outer boundary to create a region that has hollowed out sections. Using these techniques allows us to use the Area and Mass Properties tools to evaluate the information for Engineering calculations.

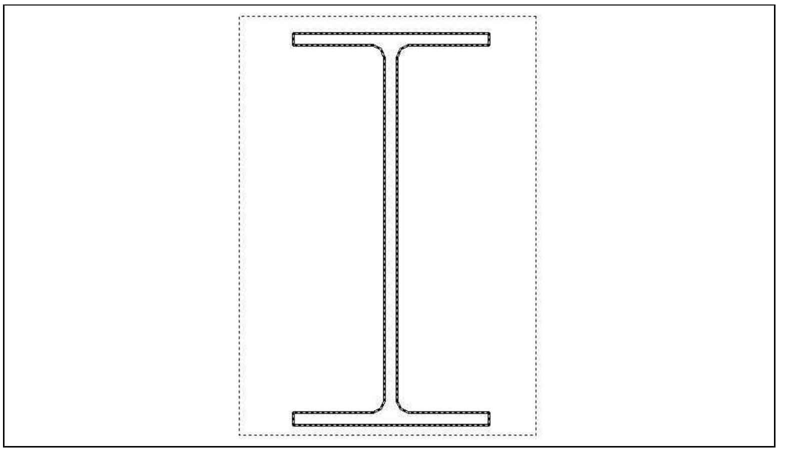

Figure 11.9 – Converting Drawing to a Region

Next, we will utilize the Massprop command on the Inquiry toolbar to give us the Moment of Inertia or bending moment for the I-beam. The number labeled as the Principle Moment and highlighted in yellow in Figure 11.10 is shown in units to the fourth power represents the capacity of the shape to resist bending when the centroid is at 0,0,0 point. For shapes designed in inches, the moment of inertia is in inches to the fourth power. For metric drawings, the answer is in millimeters to the fourth power. We know this value as the principal axis of inertia, which have their origin at the center of gravity.

In Figure 11.10, we can also observe that the volume of the shape is 0.9822 square inches and that the center of gravity is at the point 6.9413, 4.0718. We can utilize the data when doing further engineering computations.

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Area:                    0.9822
Perimeter:               15.5354
Bounding box:         X: 5.9413  --  7.9413
                      Y: 2.7018  --  6.7018
Centroid:             X: 6.9413
                      Y: 4.7018
Moments of inertia:   X: 24.1853
                      Y: 47.4893
Product of inertia:  XY: 32.0544
Radii of gyration:    X: 4.9623
                      Y: 6.9535
Principal moments and X-Y directions about centroid:
                      I: 0.1674 along [0.0000 -1.0000]
                      J: 2.4727 along [1.0000 0.0000]
```

Figure 11.10 – Mass Property Information for the I-Beam

We can see the Moment of Inertia for the I-beam is $2.4724in^4$. Our manual calculation was 2.452, which was 99% accurate. Although our accuracy was excellent when we sketched and calculated the number, the AutoCAD tool was able to give us the correct answer in seconds.

At this time, you need to complete the World Class CAD Challenge to analyze an I-beam.

*** World Class CAD Challenge 10-31 * - Open a new file in your CAD program. Make a drawing of a 4 × 2 I-beam with a web thickness of 0.125 and the flanges are 0.125 thick. The fillet radius is 0.125. Dimension the shape. Insert a border, title block and manufacturing notes. Show the center of gravity on the drawing. Include in the notes section the engineering data on the print displaying the cross sectional area, perimeter and the Moment of Inertia. Complete the drawing in 15 minutes and save the drawing as I-beam.dwg to maintain your World Class ranking.**

Continue this drill multiple times using the steps we have learned, each time completing the drawing under 15 minutes to maintain your World Class ranking

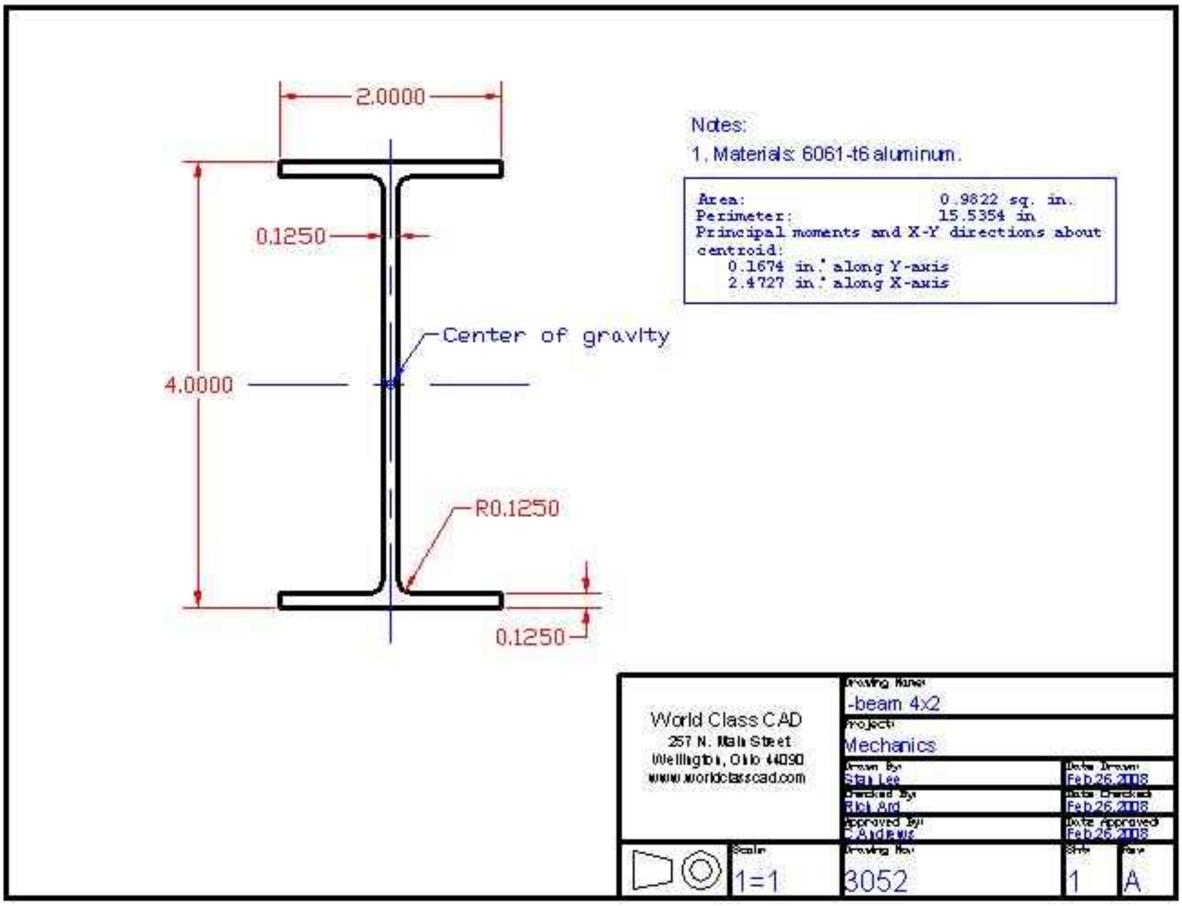

Figure 11.11 – Engineering Drawing for the 4 × 2 I-beam

Computing the Moment of Inertia for Hollow Shapes

In the next task, we will determine the Moment of Inertia of a shape that is not solid, but has an interior that we need to subtract from the outside boundary. In Figure 11.12, we will draw the exterior boundary of the custom extrusion. For our ease, we will combine all of the entities of the exterior edge to make a continuous Polyline. After completing that task, we will offset the outside perimeter 3/16 or 0.1875 to the inside. The two-dimensional shape is now complete.

To evaluate the figure and to get an accurate area and Moment of Inertia, we will convert both Polylines to Regions using the Region tool on the Draw toolbar. So, select the Region command and window both Polylines and press Enter to convert them. Now we have two separate regions.

To create a new region that removes the interior area from the exterior area, we use the Subtract tool on the Solids Editing toolbar. After picking the tool, chose the exterior perimeter first and Enter. Then pick the interior boundary and Enter. We now have a hollow region, which we can analyze using the Mass Properties tool. The Mass Property data is shown in Figure 11.13.

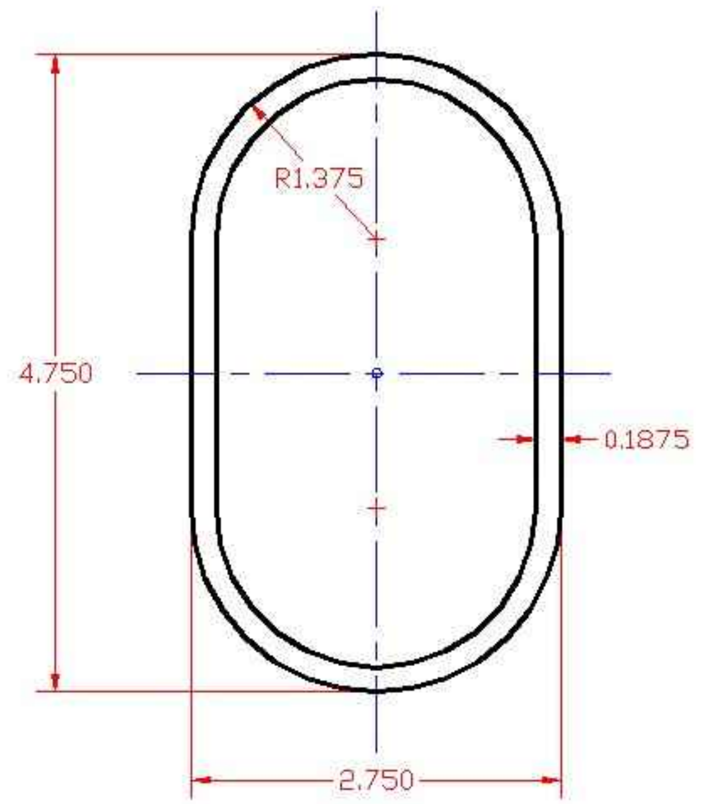

Figure 11.12 – A Custom Hollow Shape

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Area:                     2.2594
Perimeter:                24.1007
Bounding box:          X: 6.7683  --   9.5183
                       Y: 4.0013  --   8.7513
Centroid:              X: 8.1433
                       Y: 6.3763
Moments of inertia:    X: 97.3349
                       Y: 152.3090
Product of inertia:   XY: 117.3194
Radii of gyration:     X: 6.5635
                       Y: 8.2104
Principal moments and X-Y directions about centroid:
                       I: 5.4718 along [1.0000 0.0000]
                       J: 2.4790 along [0.0000 1.0000]
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Figure 11.13 – Engineering Data for the Custom Extrusion

At this time, you need to complete the World Class CAD Challenge to analyze the custom extrusion.

*** World Class CAD Challenge 10-32 * - Open a new file in your CAD program. Make a drawing of the special extrusion in Figure 11.12 with a thickness of 0.1875. Dimension the shape. Insert a border, title block and manufacturing notes. Show the center of gravity on the drawing. Include in the notes section the engineering data on the print displaying the cross sectional area, and the Moment of Inertia. Complete the drawing in 15 minutes and save the drawing as specialextrusion.dwg to maintain your World Class ranking.**

Continue this drill multiple times using the steps we have learned, each time completing the drawing under 15 minutes to maintain your World Class ranking

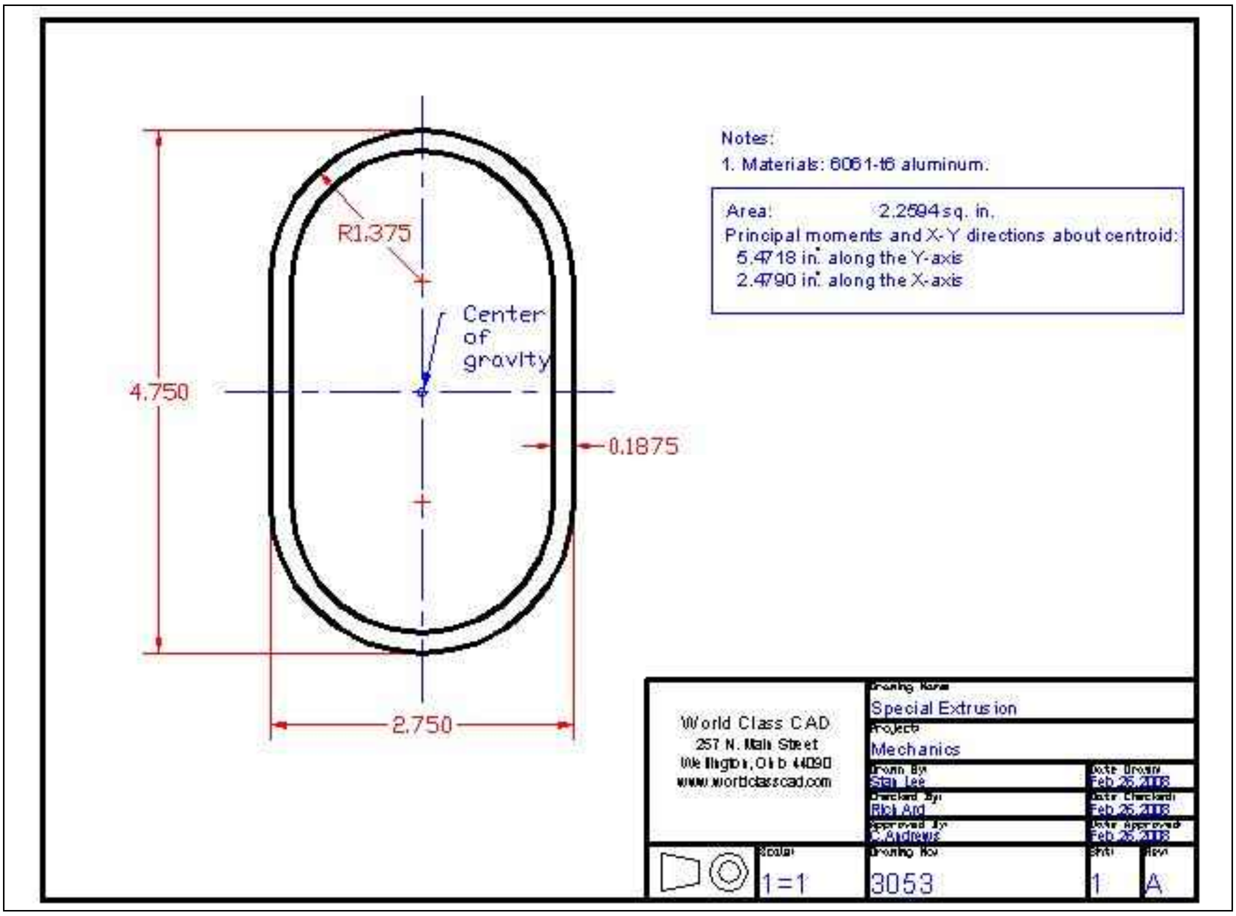

Figure 11.14 – Engineering Drawing for the Custom Extrusion

Computing the Moment of Inertia for Irregular Shapes

The next exercise is to compute the Moment of Inertia for a shape that has a complex curve. We would find the task difficult even using formula provided in any engineering handbook. A part like the one shown in Figure 11.15 would require Calculus to find the Moment of Inertia.

Using the Spline tool on the Draw toolbar, we will draw a curved line similar to the one shown in Figure 11.15. Offset the spline 1/8 of an inch and connect both splines with straight lines from endpoint to endpoint. Create a region by picking the Region tool on the Draw toolbar and select the two straight lines and the two splines. Now that the shape is a region, we can use the Mass Properties tool determine the area, perimeter and Moment of Inertia.

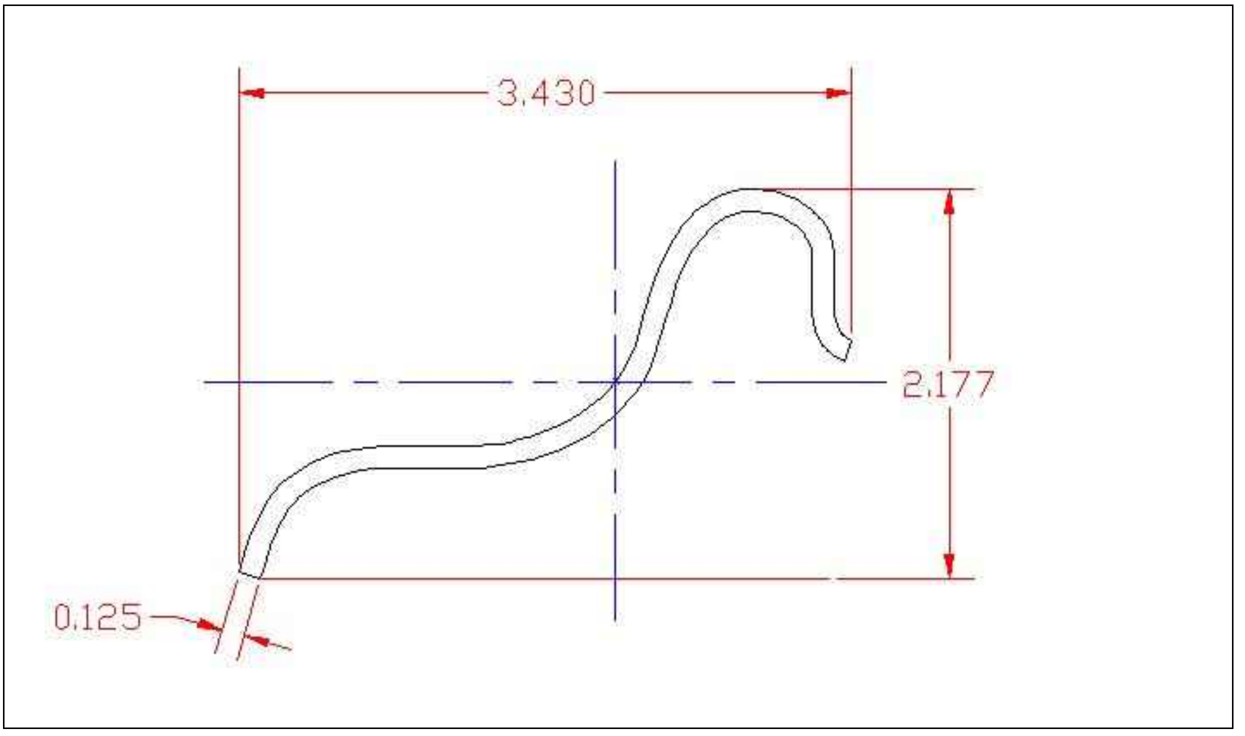

Figure 11.15 – An Irregular Shape

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Area:                       0.6377
Perimeter:                  10.3675
Bounding box:            X: 6.3241  --  9.8305
                         Y: 5.1875  --  7.4991
Centroid:                X: 8.2208
                         Y: 6.3454
Moments of inertia:      X: 25.9065
                         Y: 43.7918
Product of inertia:     XY: 33.6199
Radii of gyration:       X: 6.3739
                         Y: 8.2870
Principal moments and X-Y directions about centroid:
                         I: 0.0386 along [0.8795 0.4759]
                         J: 0.8893 along [-0.4759 0.8795]
```

Figure 11.16 – Engineering Data for the Irregular Shape

At this time, you need to complete the World Class CAD Challenge to analyze the special shape.

*** World Class CAD Challenge 10-33 * - Open a new file in your CAD program. Make a drawing of the irregular shape in Figure 11.15 with a thickness of 0.125. Dimension the shape. Insert a border, title block and manufacturing notes. Show the center of gravity on the drawing. Include in the notes section the engineering data on the print displaying the cross sectional area, the perimeter and the Moment of Inertia. Complete the drawing in 15 minutes and save the drawing as irregularshape.dwg to maintain your World Class ranking.**

Continue this drill multiple times using the steps we have learned, each time completing the drawing under 15 minutes to maintain your World Class ranking

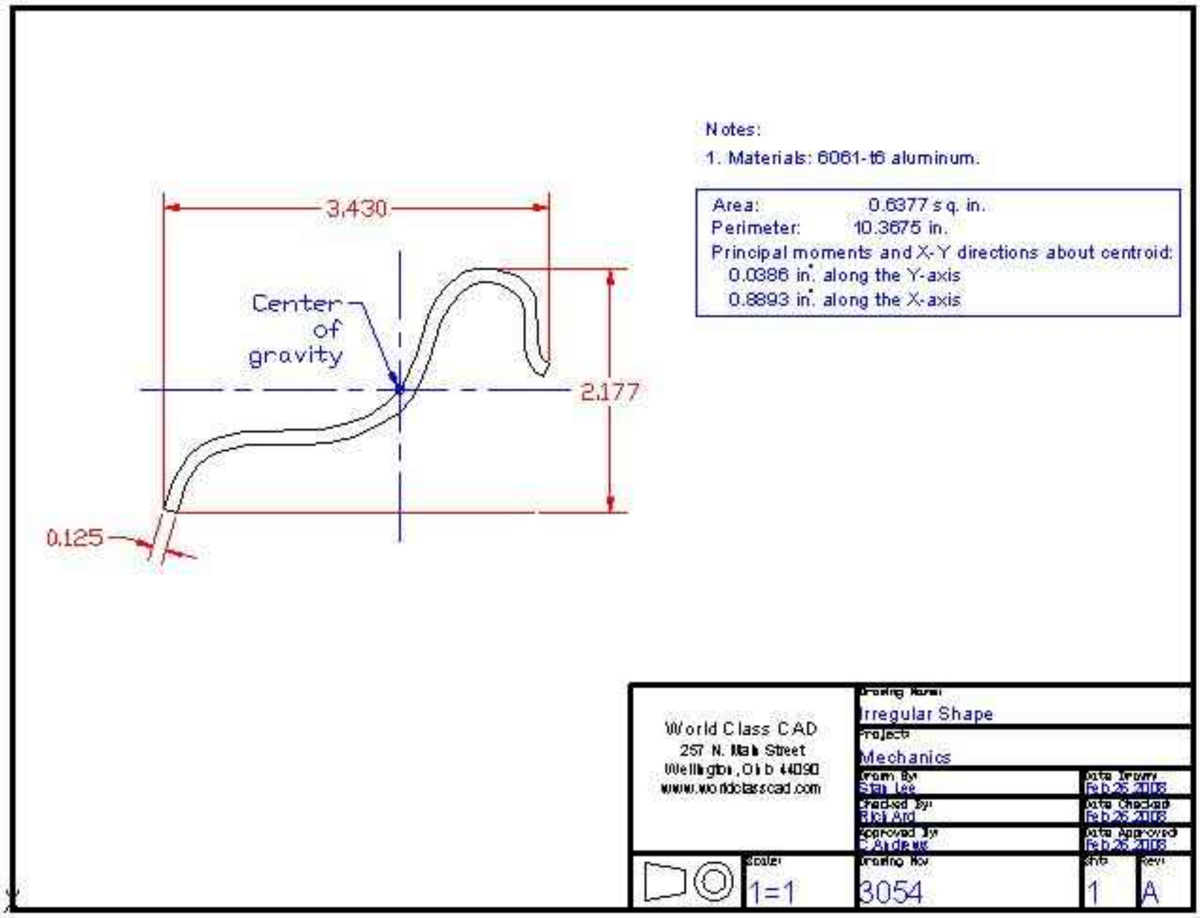

Figure 11.17 – Engineering Drawing for the Irregular Shape