Chapter 1

Basic Math Principles

In this chapter, we will learn the following to World Class CAD standards:

- ☑ **Create a Fraction**
- ☑ **Convert a Fraction to Lowest Terms**
- ☑ **Adding Fractions with a Common Denominator**
- ☑ **Adding Fractions that Do Not Have a Common Denominator**
- ☑ **Subtracting Fractions that Do Not Have a Common Denominator**
- ☑ **Multiplying Fractions that Do Not Have a Common Denominator**
- ☑ **Dividing Fractions that Do Not Have a Common Denominator**

Fractions

We have two choices to make when we have to describe a number that is not whole, such as 1, 2, 3 or 4. One choice is to produce a decimal and the other is to create a fraction. In this chapter, we will learn about fractions, which are a two number expression that have a whole number representing what we observe divided by a whole number representing the number of divisions we see that make up one unit.

We have a block of wood as shown in figure 1.1. If we were asked how many blocks we see, we would respond with:

$$1$$

We say that we see one and the number of units making a whole block is one, so the fraction is:

$$\frac{1}{1}$$

Figure 1.1 – A Whole Block

Above, we see a **fraction**, which has a **numerator**, which is the number that we observe above the line and a **denominator** the number below the line.

Now, we will cut the block exactly into two equal pieces as shown in figure 1.2. The number of pieces we observe is:

$$1$$

How many of these sized pieces make up a single unit?

$$2$$

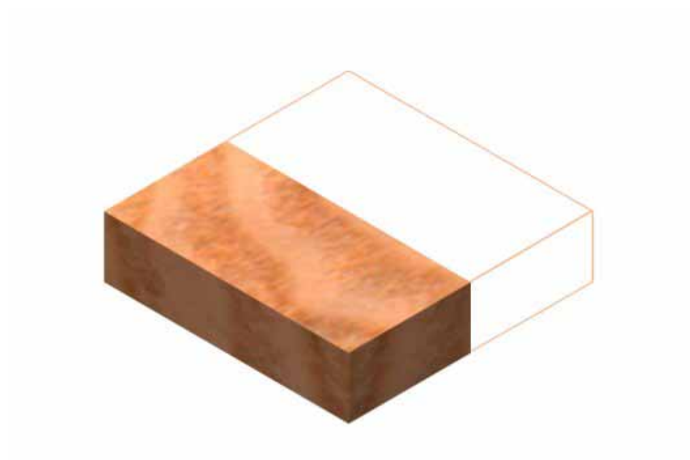


Figure 1.2 – Half of a Whole Block

We write the number that we observe in the numerator location and the number of pieces that make up a single unit in the denominator place, so the fraction will be:

$$\frac{1}{2}$$

One of the technicians cut a whole block into the "L" shaped section that we see in figure 1.3. What is the fraction of the whole that we see?

We may not have enough information to form a correct answer, so we will learn a technique to get an answer. First, we will learn about a grid.

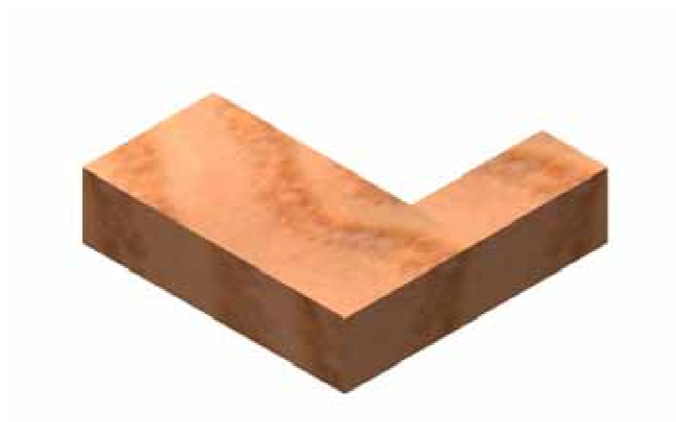


Figure 1.3 – A Portion of a Block

A **grid** is a sequence of lines that allows us to subdivide information to make a quick evaluation of what we are observing.

We scanned the wood block into our Computer Aided Design (CAD) program, and we placed a grid over the entire wood piece using a spacing equal to the width of the thin section of the "L". The grid matches perfectly. We observe ten divisions of wood so the numerator is:

$$10$$

We would need sixteen small units to fill the four by four grid, so the denominator is:

$$16$$

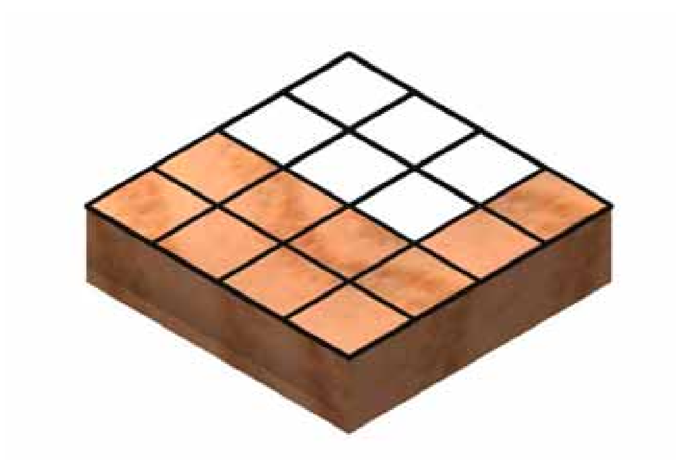
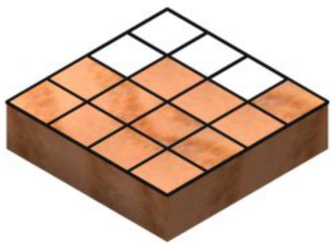
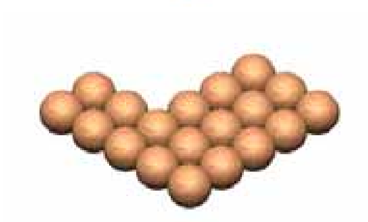
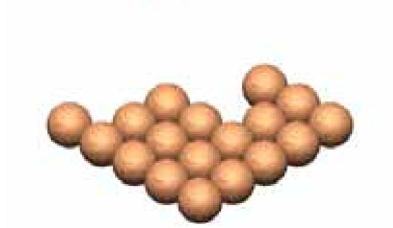


Figure 1.4 – Placing a Grid on a Block

We write the number of wood sections that we see in the grid in the numerator location of the fraction and the number of places in the grid that make a single unit in the denominator place, so the fraction will be:

$$\frac{10}{16}$$

World Class CAD Challenge 25–1: Create a fraction for each one of these images.

No.	Image	Fraction	Lowest Terms
1			
2			
3			
4			
5			

Think about it – Creating a Fraction

Answer the questions to the best of your ability. Use examples and sketches to explain your answers.

What is a fraction?	
If you have three quarters, what fraction of a dollar do you have?	
What is a numerator?	
In the fraction 2/3, what is that numerator?	
What is a denominator?	
In the fraction 5/8, what is that denominator?	
What is a grid?	
Draw a picture of a twelve pack of diet soda and place a grid on the package.	

Convert a Fraction to Lowest Terms

In our last example in the previous section of this chapter, we computed an answer of:

$$\frac{10}{16}$$

We may already know that $\frac{10}{16}$ is not in the lowest terms, but first we have to understand proportions in order to change a fraction to its numeric equivalent.

In figure 1.5, we place the four by four grid over a half block we previously used. The numerator of the fraction is eight and the denominator is sixteen, making our fraction:

$$\frac{8}{16}$$

However, we already know from previous work that the answer for this segment of block is:

$$\frac{1}{2}$$

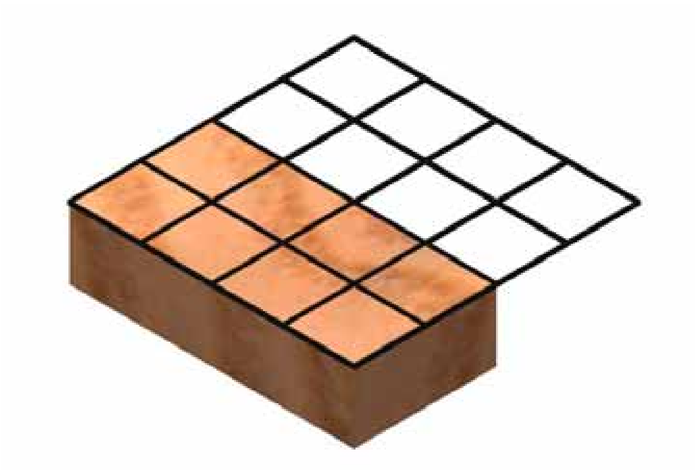


Figure 1.5 – Placing a Grid on a Block

Does that mean that $\frac{8}{16}$ equals $\frac{1}{2}$? Yes it does, because $\frac{8}{16}$ is **proportional** to $\frac{1}{2}$. Let us prove the concept mathematically by dividing the larger fraction by the same number on both the top and the bottom to see if we get one half. If we multiply or divide a fraction by the same number on the top and bottom, the number is **proportional**.

$$\frac{8 \div 2}{16 \div 2} = \frac{4}{8} \text{ then } \frac{4 \div 2}{8 \div 2} = \frac{2}{4} \text{ as well as } \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

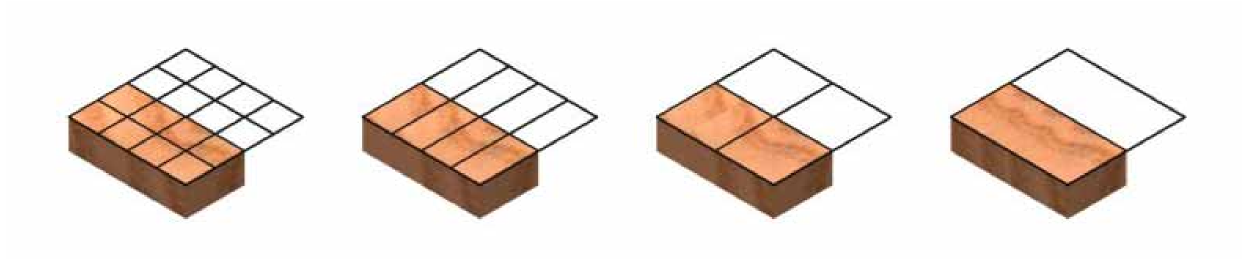


Figure 1.6 – Visually Recognizing Proportions

By dividing both the numerator and the denominator by two reduces $\frac{8}{16}$ to $\frac{4}{8}$, then $\frac{4}{8}$ to $\frac{2}{4}$, and finally $\frac{2}{4}$ to $\frac{1}{2}$. Therefore, we now know that $\frac{8}{16}=\frac{4}{8}=\frac{2}{4}=\frac{1}{2}$. We could have easily gone in the other direction, multiplying both the numerator and the denominator by two never completing the task since both parts of the fraction would go to infinity.

To answer the question from the beginning of the section $\frac{10 \div 2}{16 \div 2}=\frac{5}{8}$ and now the fraction is in the lowest terms.

World Class CAD Challenge 25–2: Convert these fractions to the lowest terms.

No.	Problem	Workspace	Answer
1	$\frac{12}{20}$		
2	$\frac{14}{21}$		
3	$\frac{15}{25}$		
4	$\frac{13}{18}$		
5	$\frac{3}{9}$		
6	$\frac{16}{64}$		
7	$\frac{1024}{65536}$		
8	$\frac{10,000}{1,000,000}$		

Think about it – Lowest Terms

Answer the questions to the best of your ability. Use examples and sketches to explain your answers.

When is a fraction in lowest terms?	
Create a large fraction and reduce the fraction to the lowest terms	
When is one fraction proportional to another fraction?	

Adding Fractions with a Common Denominator

To add two or more factions together is quite easy when they have the same denominator, such as:

$$\frac{3}{8}+\frac{5}{8}$$

When the denominators are equal, we just add the numerators together as we see below.

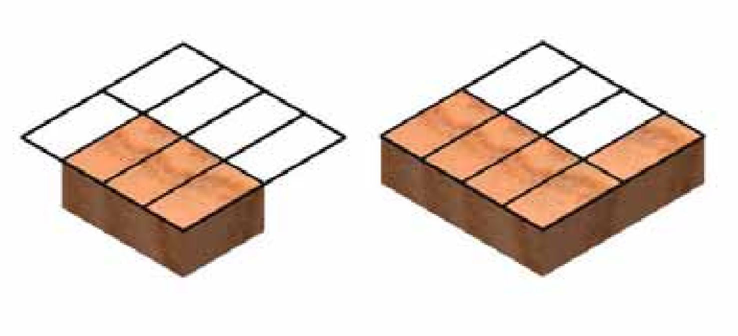


Figure 1.7 – Adding Fractions Together

$$\frac{3}{8}+\frac{5}{8}=\frac{8}{8}=1$$

The answer for $\frac{3}{8}$ plus $\frac{5}{8}$ is $\frac{8}{8}$ or in lowest terms, 1.

Sometimes, fractions that are added together result in an improper format, where the numerator is larger than the denominator, such as in:

$$\frac{5}{8}+\frac{7}{8}$$

When they are added as one, the answer can be modified.

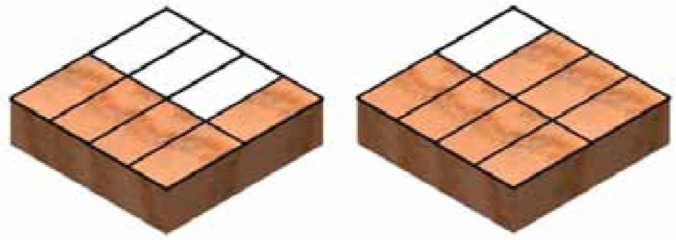


Figure 1.8 – Adding to a Mixed Number

$$\frac{5}{8}+\frac{7}{8}=\frac{12}{8}$$

This format $\frac{12}{8}$ is referred to as an **improper fraction**. Some professionals will have us record our answer as is, but others will want us to change the **improper fraction** to a **mixed number**. To find the mixed number for $\frac{12}{8}$, divide the numerator by the denominator and place the remainder over the denominator.

$$\begin{array}{r} 1\ \text{R4} \\ 8\,\overline{)\,12} \\ \underline{8} \\ 4 \end{array}$$

When we divide twelve by eight, we get one remainder four. As shown below, the answer can be $1\frac{4}{8}$ or written as $1\frac{1}{2}$, which is in lowest terms.

$$\frac{12}{8}=1\frac{4}{8}=1\frac{1}{2}$$

World Class CAD Challenge 25–3: Add these fractions and convert to the lowest terms.

No.	Problem	Workspace	Answer
1	$\frac{2}{3}+\frac{1}{3}$		

2	$\frac{3}{8}+\frac{7}{8}$		
3	$\frac{4}{9}+\frac{7}{9}$		
4	$\frac{15}{25}+\frac{45}{50}$		
5	$\frac{10}{14}+\frac{7}{17}$		
6	$\frac{81}{100}+\frac{26}{100}$		
7	$2\frac{3}{8}+\frac{1}{8}$		
8	$\frac{3}{4}+2\frac{1}{4}$		

Think about it – Adding Fractions with Common Denominators

Answer the questions to the best of your ability. Use examples and sketches to explain your answers.

Can you add a fraction with a common denominator?	
What is an improper fraction?	
What is a mixed number?	
How do you change an improper fraction into a mixed number?	

Adding Fractions that Do Not Have a Common Denominator

When we are adding two fractions that do not have a common denominator and we cannot easy change one of the fractions to the other's denominator, follow these steps that we will learn in this example:

$$\frac{2}{5}+\frac{1}{6}$$

The common denominator for the above problem is 5 x 6 = 30. We arrive at this number by multiplying the denominator of the first fraction (5) and the denominator of the second fraction (6).

Write down the new problem as:

$$\frac{\quad}{30}+\frac{\quad}{30}$$

Next, we need to find the numerator that is written over the 30 for the first fraction, so we write another expression as shown below:

$$\frac{2}{5}=\frac{\quad}{30}$$

Now 5 times what number equals 30? Six, **right**! Now, whatever number we multiply on the bottom, go ahead and multiply on the top. So 2 x 6 = 12 as we can see below.

$$\frac{2}{5}\times\frac{6}{6}=\frac{12}{30}$$

To find the numerator that is written over the 30 in the second fraction, write:

$$\frac{1}{6}=\frac{\quad}{30}$$

At this time, 6 times what number equals 30? Five, **correct**! Again, when we multiply on the bottom, go ahead and multiply on the top. So 1 x 5 = 5 as we can see below.

$$\frac{1}{6}\times\frac{5}{5}=\frac{5}{30}$$

Place the new numerators over the denominators on your paper and now we solve the problem.

$$\frac{12}{30}+\frac{5}{30}=\frac{17}{30}$$

12 + 5 = 17 as the denominator stays as 30. The answer is $\frac{17}{30}$.

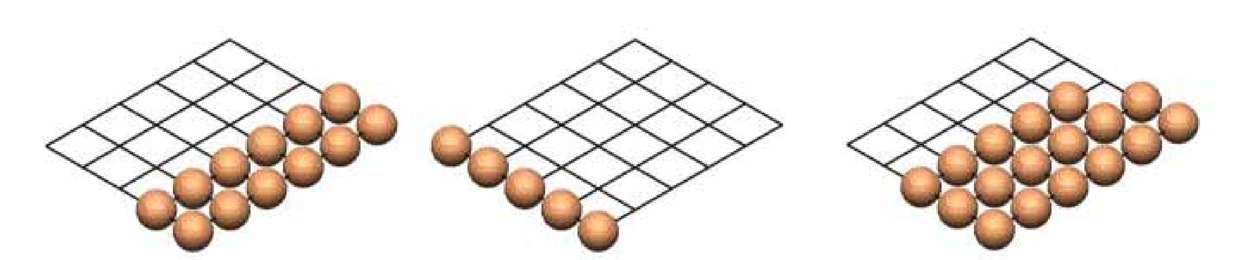


Figure 1.9 – Adding Fractions Once Converted They Have a Common Denominator

World Class CAD Challenge 25–4: Add these fractions and convert to the lowest terms.

1	$\frac{1}{3}+\frac{1}{2}$	**2**	$\frac{2}{3}+\frac{1}{4}$
3	$\frac{2}{5}+\frac{1}{6}$	**4**	$\frac{3}{8}+\frac{1}{4}$
5	$\frac{3}{4}+\frac{5}{6}$	**6**	$\frac{3}{5}+\frac{4}{7}$
7	$\frac{3}{10}+\frac{3}{5}$	**8**	$\frac{7}{12}+\frac{5}{6}$
9	$\frac{7}{8}+\frac{5}{6}$	**10**	$\frac{2}{3}+\frac{4}{7}$

Think about it – Adding Fractions that Do Not Have a Common Denominators

Answer the questions to the best of your ability. Use examples and sketches to explain your answers.

How do you make denominators common?	

Subtracting Fractions that Do Not Have a Common Denominator

When we are subtracting two fractions that do not have a common denominator and we cannot easy change one of the fractions to the other's denominator, follow these steps on this problem:

$$\frac{4}{7} - \frac{2}{5}$$

The common denominator for the above problem is 7 x 5 = 35. We arrive at this number by multiplying the denominator of the first fraction (7) and the denominator of the second fraction (5).

Write down the new expression as:

$$\frac{\quad}{35} - \frac{\quad}{35}$$

To find the numerator for the first fraction, write:

$$\frac{4}{7} = \frac{\quad}{35}$$

Now 7 times what number equals 35? Five, **exactly**! To keep the fraction proportional, multiply the same number on the bottom and on the top. So 4 x 5 = 20 as we can see below.

$$\frac{4}{7} \times \frac{5}{5} = \frac{20}{35}$$

To find the numerator for the second fraction, write:

$$\frac{2}{5} = \frac{\quad}{35}$$

At this time, 5 times what number equals 35? Seven, **yes**! Now, whenever we multiply on the bottom, go ahead and multiply on the top. So 2 x 7 = 14 as we can see below.

$$\frac{2}{5} \times \frac{7}{7} = \frac{14}{35}$$

Place the new numerators over the denominators on the paper and now we solve the problem.

$$\frac{20}{35} - \frac{14}{35} = \frac{6}{35}$$

20 – 14 = 6 and the denominator stays as 35. The answer is $\frac{6}{35}$.

World Class CAD Challenge 25–5: Subtract these fractions and convert to the lowest terms.

1	$\frac{1}{2} - \frac{1}{5}$	**2**	$\frac{3}{4} - \frac{1}{3}$
3	$\frac{9}{10} - \frac{3}{4}$	**4**	$\frac{7}{8} - \frac{2}{5}$
5	$\frac{4}{9} - \frac{1}{5}$	**6**	$\frac{2}{7} - \frac{3}{12}$
7	$\frac{6}{7} - \frac{2}{3}$	**8**	$\frac{7}{12} - \frac{2}{5}$
9	$\frac{1}{2} - \frac{3}{10}$	**10**	$\frac{4}{5} - \frac{3}{4}$

Think about it – Subtracting Fractions that Do Not Have a Common Denominators

Answer the questions to the best of your ability. Use examples and sketches to explain your answers.

What differences are there between adding and subtracting fractions?	

Multiplying Fractions that Do Not Have a Common Denominator

When we are multiplying two or more fractions, we no longer need to have common denominators in all of the numbers, so many people feel that these problems are easier to solve than adding and subtracting fractions. Follow these steps when multiplying fractions.

$$\frac{3}{4} \times \frac{2}{5}$$

Multiply the numerators across the top, so in this problem we compute 3 x 2 = 6

Then multiply the denominators across the bottom, so in this problem we compute 4 x 5 = 20

So, write down the new problem as:

$$\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$$

To write the answer in the lowest terms, we find a divisor that is common to both numerator and the denominator. In this answer $\frac{6}{20}$, the number 2 goes into both 6 and 20. Divide the numerator and the denominator by 2 to get the result as $\frac{3}{10}$.

$$\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$$

World Class CAD Challenge 25–6: Multiply these fractions and convert to the lowest terms.

1	$\frac{1}{3} \times \frac{1}{2}$	2	$\frac{2}{3} \times \frac{1}{4}$
3	$\frac{2}{5} \times \frac{5}{6}$	4	$\frac{3}{8} \times \frac{1}{4}$
5	$\frac{3}{4} \times \frac{5}{6}$	6	$\frac{3}{5} \times \frac{4}{7}$
7	$\frac{7}{10} \times \frac{4}{5}$	8	$\frac{7}{12} \times \frac{5}{6}$
9	$\frac{7}{8} \times \frac{5}{6}$	10	$\frac{2}{3} \times \frac{4}{7} \times \frac{1}{2}$

Think about it – Multiplying Fractions that Do Not Have a Common Denominators

Answer the questions to the best of your ability. Use examples and sketches to explain your answers.

Why is multiplying fractions so easy?	
Do we have to have common denominators to multiply fractions?	

Dividing Fractions that Do Not Have a Common Denominator

When we are dividing with fractions, again, we no longer need to have common denominators in all of the numbers, so people feel that these problems are easier to solve than adding and subtracting fractions. Follow these steps when dividing:

$$\frac{3}{4} \div \frac{2}{3}$$

Rewrite the problem by changing the division symbol to multiplication and modifying the second fraction $\frac{2}{3}$ by writing its **reciprocal** or flipping the fraction to $\frac{3}{2}$

So, write down the new problem as:

$$\frac{3}{4} \times \frac{3}{2}$$

Multiply the numerators across the top, so in this problem we compute 3 x 3 = 9

Then multiply the denominators across the bottom, so in this problem we compute 4 x 2 = 8

So, write down the new problem as:

$$\frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$$

To write the answer as a mixed fraction, we divide the denominator 8 into the numerator 9. We get one with a remainder of one to obtain the result of $1\frac{1}{8}$.

World Class CAD Challenge 25–6: Dividing these fractions and convert to the lowest terms.

1	$\frac{1}{3} \div \frac{1}{2}$	**2**	$\frac{2}{3} \div \frac{1}{4}$
3	$\frac{2}{5} \div \frac{5}{6}$	**4**	$\frac{3}{8} \div \frac{3}{4}$

5	$\frac{3}{4} \div \frac{5}{6}$	**6**	$\frac{3}{5} \div \frac{4}{7}$
7	$\frac{7}{10} \div \frac{4}{5}$	**8**	$\frac{7}{12} \div \frac{5}{6}$
9	$\frac{7}{8} \div \frac{5}{6}$	**10**	$\frac{2}{3} \div \frac{4}{7}$

Think about it – Dividing Fractions that Do Not Have a Common Denominators

Answer the questions to the best of your ability. Use examples and sketches to explain your answers.

What is a reciprocal?	
What is the reciprocal of 2/3?	
When is multiplying and dividing fractions similar?	

For those of us who are already familiar with fractions, this chapter was a good review. For anyone who has forgotten various techniques in creating, converting, adding, subtracting, multiplying and dividing fractions, this chapter reviews and allows one to practice many problems dealing with fractions. The next chapter in this series will deal with converting from one unit to another.